



# Last lecture (5)

- Solar wind
  - magnetic structure
- Ionosphere
  - index of refraction
  - reflection of radio waves
  - particle drift motion in magnetized plasma

# Today's lecture (6)

- Electrical conductivity in ionosphere
- Magnetosphere, introduction
- Magnetospheric size (standoff distance)
- Particle motion in the magnetosphere



# Today

Activity	Date	Time	Room	Subject	Litterature
L1	28/8	15-17	Q21	Course description, Introduction, The Sun 1	<b>CGF</b> Ch 1.1,1.2, 1.4, 5, (p 110-113), 6.3
L2	29/8	13-15	Q2	The Sun 2, Plasma physics 1	<b>CGF</b> Ch 1.3, 5 (p 114-121)
L3	4/9	10-12	E2	Solar wind, The ionosphere and atmosphere 1, Plasma physics 2	<b>CGF</b> Ch 6.1, 2, 3.1-3.2, 3.5, <b>LL</b> Ch III, Extra material
T1	6/9	8-10	Q21	Mini-group work 1	
L4	6/9	15-17	Q2	The ionosphere 2, Plasma physics 3	<b>CGF</b> Ch 3.4, 3.7, 3.8
T2	10/9	15-17	Q21	Mini-group work 2	
L5	11/9	10-12	E3	The Earth's magnetosphere 1, Plasma physics 4	<b>CGF</b> 4-1-4.3, <b>LL</b> Ch I, II, IV.A
T3	17/9	8-10	Q21	Mini-group work 3	
L6	18/9	13-15	Q33	The Earth's magnetosphere 2, Other magnetospheres	<b>CGF</b> Ch 4.6-4.9, <b>LL</b> Ch V.
L7	19/9	13-15	Q2	Aurora, Measurement methods in space plasmas and data analysis 1	<b>CGF</b> Ch 4.5, 10, <b>LL</b> Ch VI, Extra material
T4	24/9	8-10	Q2	Mini-group work 4	
L8	24/9	15-17	V3	Space weather and geomagnetic storms	<b>CGF</b> Ch 4.4, <b>LL</b> Ch IV.B-C, VII.A-C
T5	2/10	8-10	Q31	Mini-group work 5	
L9	2/10	13-15	Q2	Alfvén waves, Interstellar and intergalactic plasma, Cosmic radiation	<b>CGF</b> Ch 7-9, Extra material
T6	8/10	15-17	Q21		
L10	9/10	10-12	Q2	Guest Lecture by Swedish astronaut Christer Fuglesang	
Written examination	16/10	14-19	L21, L22, L31		

# **EF22445 Space Physics II**

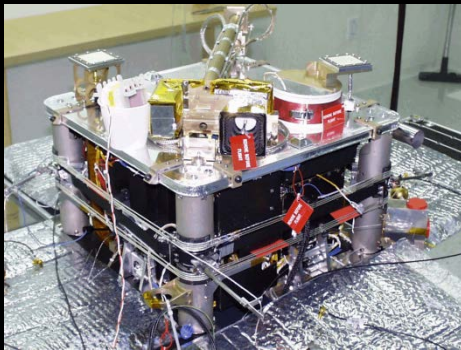
## **7.5 ECTS credits, P2**

- shocks and boundaries in space
- solar wind interaction with magnetized and unmagnetized bodies
- reconnection
- sources of magnetospheric plasma
- magnetospheric and ionospheric convection
- auroral physics
- storms and substorms
- global oscillations of the magnetosphere

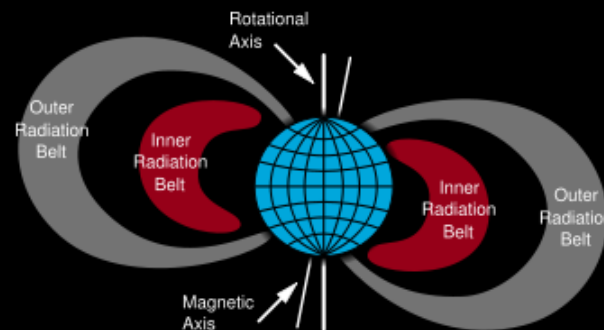
# Courses at the Alfvén Laboratory

## EF2260 SPACE ENVIRONMENT AND SPACECRAFT ENGINEERING , 6 ECTS credits, period 2

- environments spacecraft may encounter in various orbits around the Earth, and the constraints this places on spacecraft design
- basic operation principles underlying the thermal control system and the power systems in spacecraft
- measurements principles in space



*The Astrid-2 satellite*



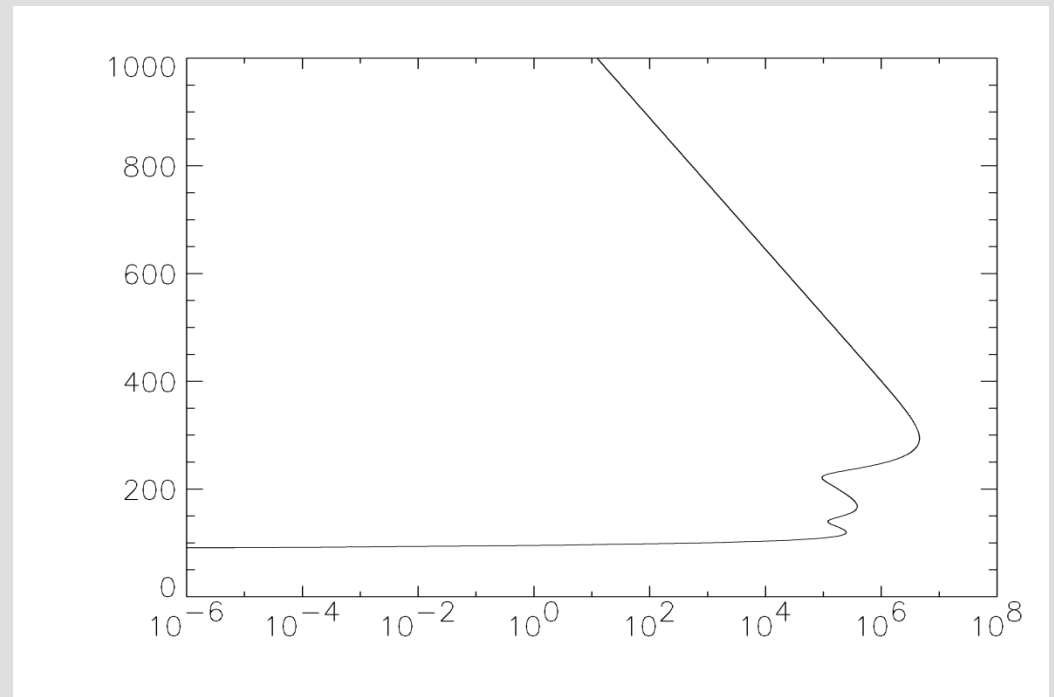
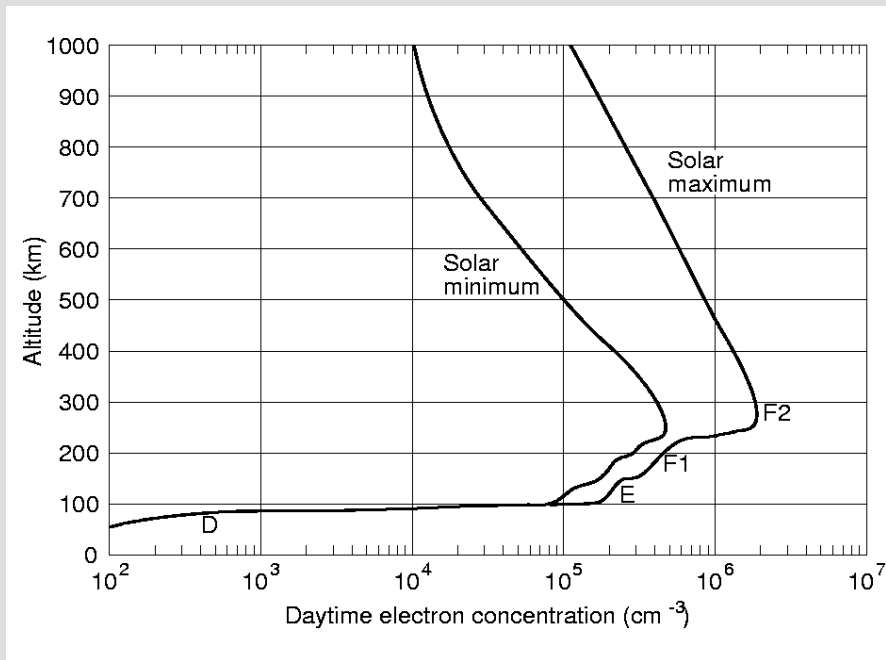
*Radiation environment in near-earth space*

### Projects:

- Design power supply for spacecraft
- Study of radiation effects on electronics

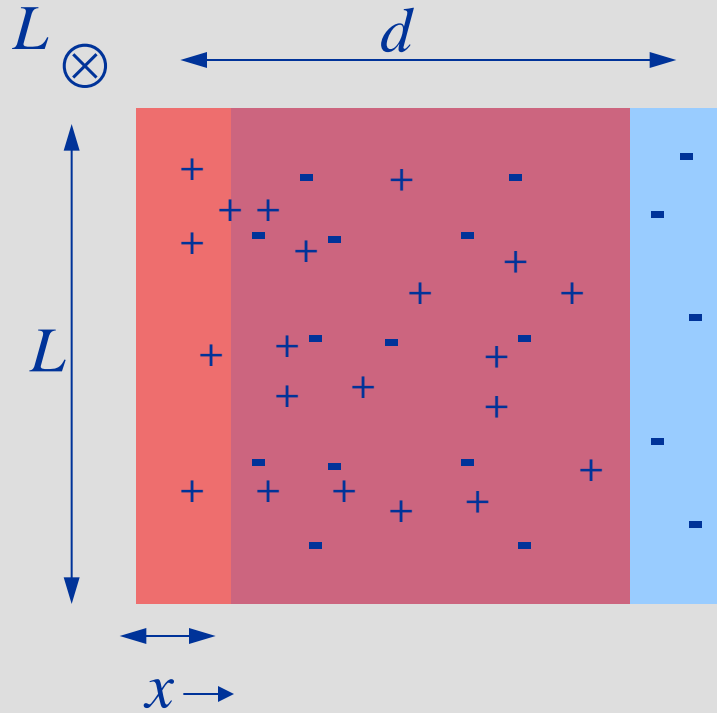
# Measurements

"E" + "F1" + "F2"



# Ionospheric layers

Layer	D	E	F <sub>1</sub>	F <sub>2</sub>
Altitude (km)	60-85	85-140	140-200	200 - ca 1500
Nighttime electron density (cm <sup>-3</sup> )	<10 <sup>2</sup>	2 · 10 <sup>3</sup>	—	2 - 5 · 10 <sup>5</sup>
Daytime electron density (cm <sup>-3</sup> )	10 <sup>3</sup>	1 - 2 · 10 <sup>5</sup>	2 - 5 · 10 <sup>5</sup>	0.5 - 2 · 10 <sup>6</sup>
Ion species	NO <sup>+</sup> O <sub>2</sub> <sup>+</sup>	NO <sup>+</sup> O <sub>2</sub> <sup>+</sup>	NO <sup>+</sup> O <sub>2</sub> <sup>+</sup> O <sup>+</sup>	O <sup>+</sup> He <sup>+</sup> H <sup>+</sup>
Cause of ionization	Lyman α (1215 Å) + cosmic radiation	Lyman β (1025 Å) X-rays	UV	UV

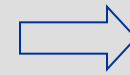


$$F = m_e a$$

$$E = \frac{\sigma}{\epsilon_0}$$

$$F = -eE$$

$$\sigma = en_e x$$



$$-\frac{n_e e^2 x}{\epsilon_0 m_e} = \frac{d^2 x}{dt^2}$$

$$x = \sin(\omega_{pe} t)$$

$$\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$$

# Index of refraction for electromagnetic waves in a plasma (corrected)

$$(1) \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$(2) \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$(3) \quad \mathbf{j} = -en_e \mathbf{v}_e$$

$$(4) \quad m_e \frac{\partial \mathbf{v}_e}{\partial t} = -e\mathbf{E}$$

Assume all quantities vary sinusoidally, with frequency  $\omega$ , e.g.:

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

$$(1) \Rightarrow \nabla \times (\nabla \times \mathbf{E}) = -\nabla \times \frac{\partial \mathbf{B}}{\partial t}$$

$$(2) \Rightarrow \nabla \times \frac{\partial \mathbf{B}}{\partial t} = \mu_0 \frac{\partial \mathbf{j}}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\therefore \nabla \times (\nabla \times \mathbf{E}) = +\mu_0 \frac{\partial}{\partial t} (en_e \mathbf{v}_e) - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$\Rightarrow$

$$\nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = +\mu_0 en_e \frac{\partial \mathbf{v}_e}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$\Rightarrow$



# Index of refraction for electromagnetic waves in a plasma (corrected)

~~$$ik(\mathbf{k} \cdot \mathbf{E}) - k^2 \mathbf{E} = \mu_0 (-i\omega) en_e \mathbf{v}_e - \frac{1}{c^2} (-i\omega)^2 \mathbf{E}$$~~

*Does not represent E.M. wave*

(4)  $\Rightarrow$

$$k^2 \mathbf{E} = -\mu_0 en_e \frac{e\mathbf{E}}{m_e} + \frac{1}{c^2} \omega^2 \mathbf{E}$$

$\Rightarrow$

$$c^2 k^2 = -c^2 \frac{\mu_0 n_e e^2}{m_e} + \omega^2 = \frac{-1}{\cancel{\mu_0 \epsilon_0}} \frac{\cancel{\mu_0} n_e e^2}{m_e} + \omega^2$$

$$\therefore \omega^2 = c^2 k^2 + \omega_p^2$$

$$n^2 = \frac{c^2}{v_{ph}^2} = \frac{c^2 k^2}{\omega^2} = \frac{\omega^2 - \omega_p^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$

$\therefore$

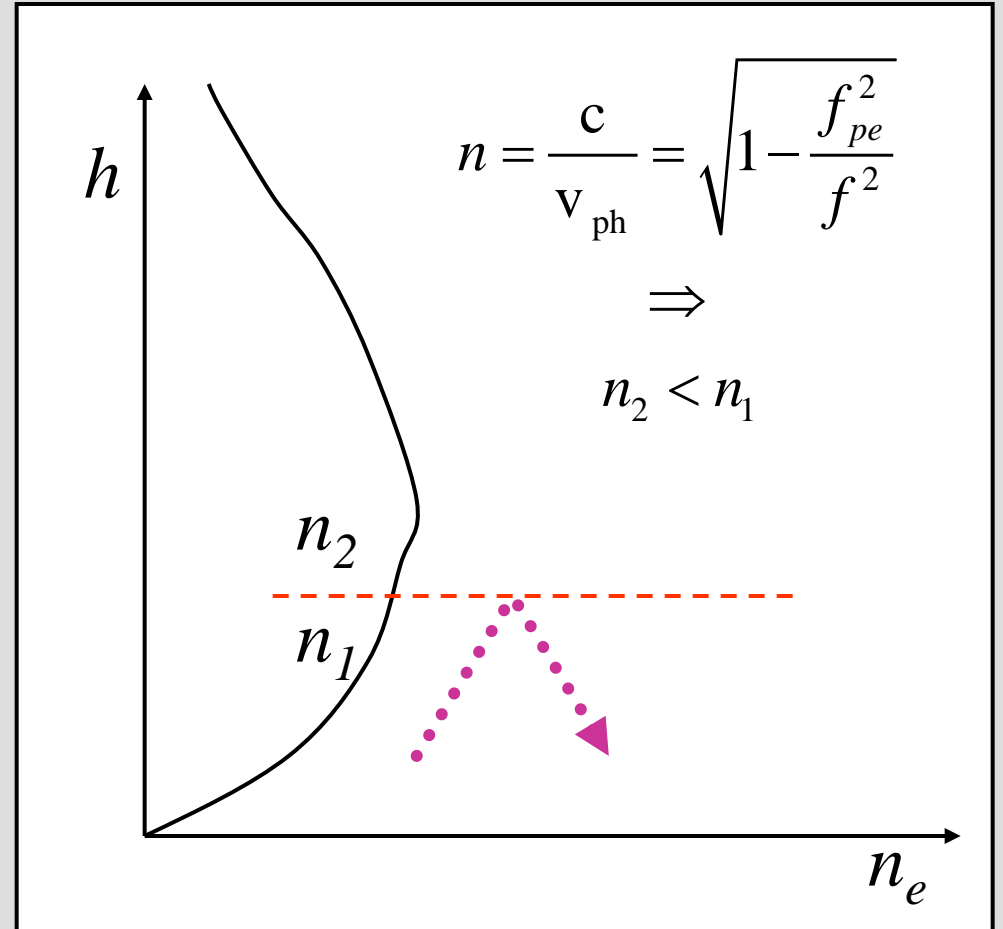
$$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} = \sqrt{1 - \frac{f_p^2}{f^2}}$$

# Where does the total reflection take place?

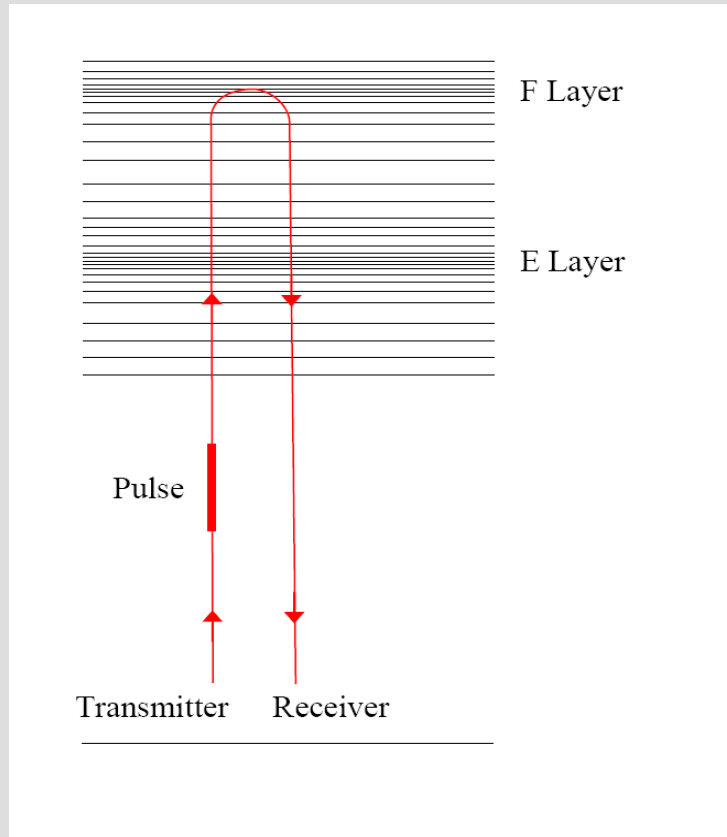
Large gradient when

$$f \approx f_{pe}$$

Higher frequencies  $\rightarrow$  higher  $f_{pe}(n_e)$



# Ionosonde



The pulse will be reflected where

$$f = f_{pe}$$

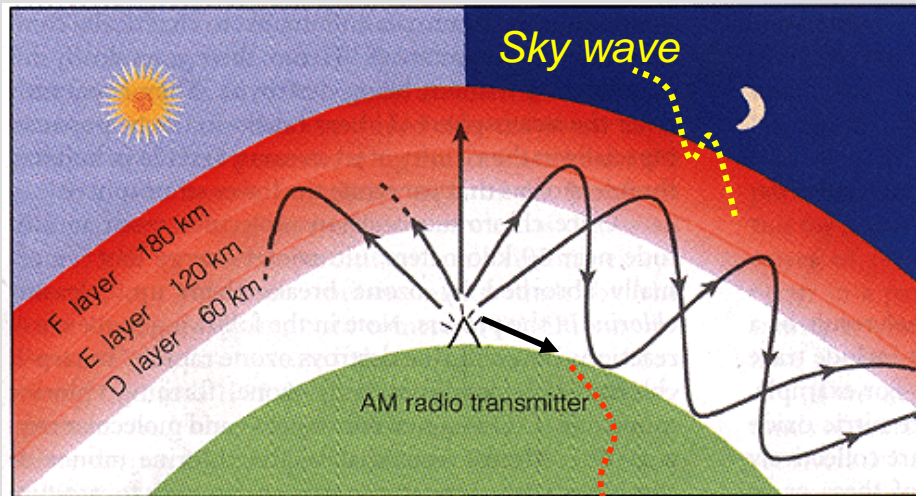
The altitude will be determined by

$$2h = ct$$

Where  $t$  is the time between when the pulse is sent out and the registered again.

# Reflection of radio waves

*F2-layer during night:*

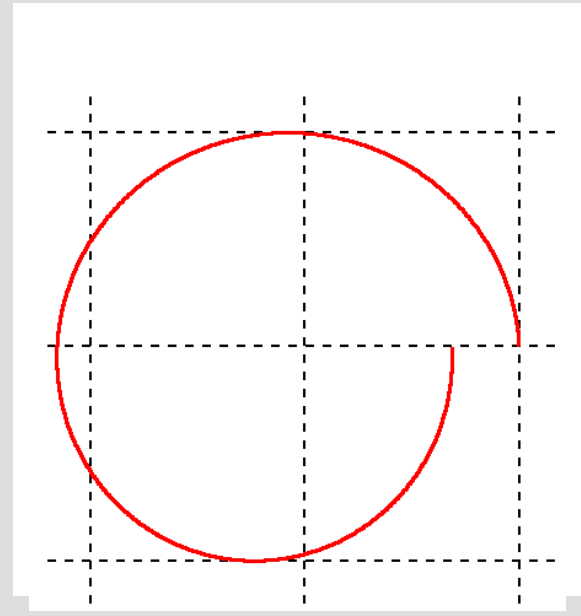
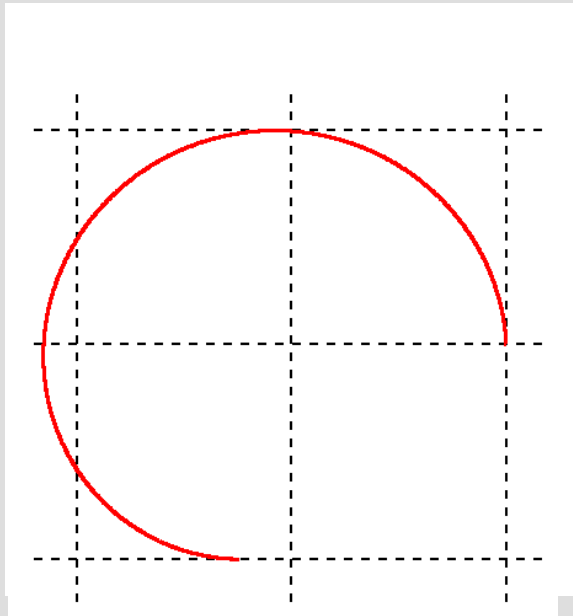
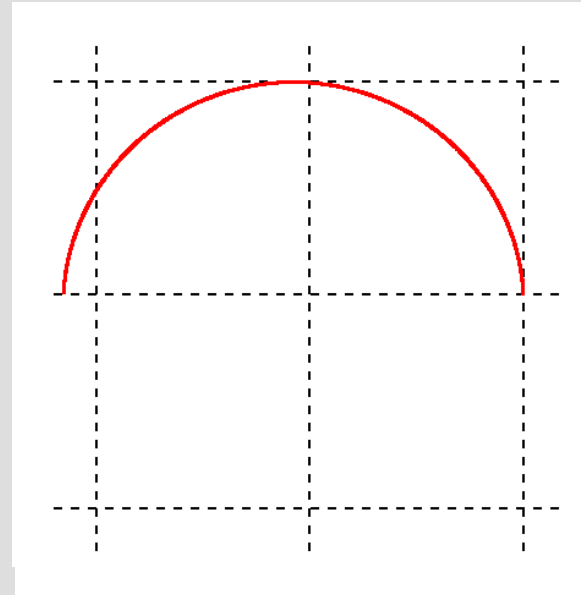
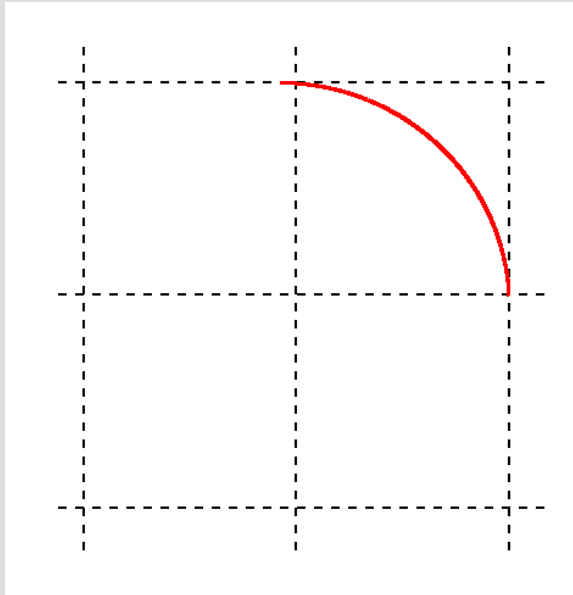


*Ground wave*

$$n_e = 5 \cdot 10^{11} \text{ m}^{-3} \Rightarrow$$

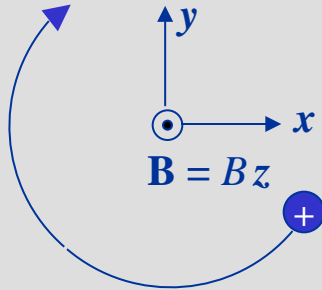
$$f_{pe} = 10^7 \text{ Hz} = 10 \text{ MHz}$$

= HF/short wave



# Drift motion

Consider a charged particle in a magnetic field.



Assume an electric field in the x-z plane:

$$\mathbf{E} = (E_x, 0, E_z)$$

$$m \frac{d\mathbf{v}}{dt} = q(\mathbf{v} \times \mathbf{B} + \mathbf{E}) \implies$$

$$\left\{ \begin{array}{l} m \frac{dv_x}{dt} = qv_y B + qE_x \\ m \frac{dv_y}{dt} = -qv_x B \\ m \frac{dv_z}{dt} = qE_z \end{array} \right. \quad \text{Constant acceleration along } z$$



$$\left\{ \begin{array}{l} \frac{d^2 v_x}{dt^2} = \frac{qB}{m} \frac{dv_y}{dt} = \omega_g \frac{dv_y}{dt} = -\omega_g^2 v_x \\ \frac{d^2 v_y}{dt^2} = -\frac{qB}{m} \frac{dv_x}{dt} = -\omega_g \frac{dv_x}{dt} = -\omega_g^2 v_y - \frac{q^2 B}{m^2} E_x \end{array} \right.$$



# Drift motion

$$\begin{cases} \frac{d^2 v_x}{dt^2} = \frac{qB}{m} \frac{dv_y}{dt} = \omega_g \frac{dv_y}{dt} = -\omega_g^2 v_x \\ \frac{d^2 v_y}{dt^2} = -\frac{qB}{m} \frac{dv_x}{dt} = -\omega_g \frac{dv_x}{dt} = -\omega_g^2 v_y - \frac{q^2 B}{m^2} E_x \end{cases}$$

∴

$$\begin{cases} \frac{d^2 v_x}{dt^2} - \omega_g^2 v_x \\ \frac{d^2 \left( v_y + \frac{E_x}{B} \right)}{dt^2} = -\omega_g^2 \left( v_y + \frac{E_x}{B} \right) \end{cases}$$



$$\begin{cases} v_x = v_{\perp} e^{i\omega_g t + \delta_x} \\ v_y = -\frac{E_x}{B} + v_{\perp} e^{i\omega_g t + \delta_y} \end{cases}$$

Average over a gyro period:

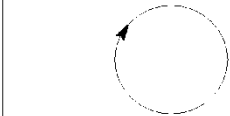
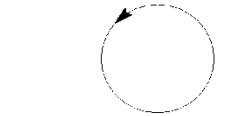
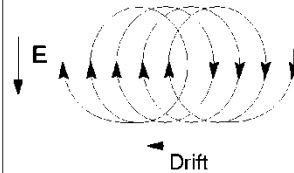
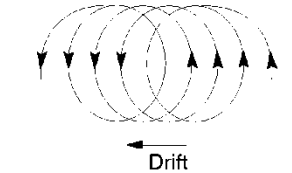
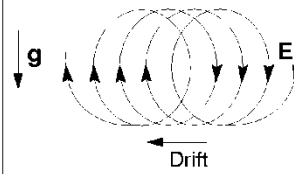
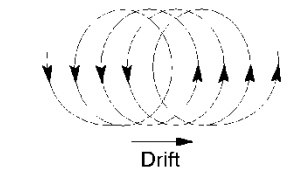
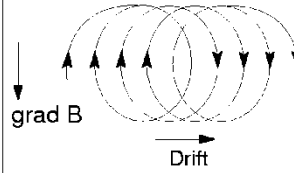
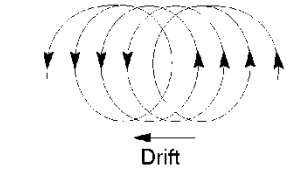
$$v_{drift,y} = -\frac{E_x}{B} = -\frac{E_x B_z}{B^2} = \frac{(\mathbf{E} \times \mathbf{B})_y}{B^2}$$

In general:

$$\mathbf{v}_{drift} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} = \frac{q\mathbf{E} \times \mathbf{B}}{qB^2} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

# Drift motion

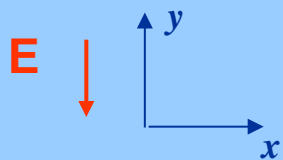
$$\mathbf{u}_{drift} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

	Positive particles	Negative particles
Homogeneous magnetic field No disturbing force $\mathbf{F} = 0$		
Homogeneous magnetic field Homogeneous electric field $\mathbf{F} = q\mathbf{E}$		
Homogeneous magnetic field Gravitation $\mathbf{F} = m\mathbf{g}$		
Inhomogeneous magnetic field $\mathbf{F} = -\mu \text{grad } \mathbf{B}$		





Suppose you apply an electric field  $\mathbf{E}$  in the direction showed in the figure, and that one electron and one ion (charge  $-e$  and  $e$ ) is present. What will the resulting current be?



$$\mathbf{I} \equiv e\mathbf{u}_i - e\mathbf{u}_e$$

Yellow

$$\mathbf{I} = -e \frac{E}{B} \hat{\mathbf{x}}$$

Blue

$$\mathbf{I} = 0$$

Red

$$\mathbf{I} = \frac{1}{2}e \frac{E}{B} \hat{\mathbf{x}} - \frac{1}{2}e \frac{E}{B} \hat{\mathbf{y}}$$

Green

$$\mathbf{I} = e \frac{E}{B} \hat{\mathbf{y}}$$

$$\mathbf{u} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

	Positive particles	Negative particles
Homogeneous magnetic field No disturbing force $\mathbf{F} = 0$		
Homogeneous magnetic field Homogeneous electric field $\mathbf{F} = q\mathbf{E}$		
Homogeneous magnetic field Gravitation $\mathbf{F} = m\mathbf{g}$		
Inhomogeneous magnetic field $\mathbf{F} = -\mu \text{grad } B$		

$$\mathbf{I} \equiv e\mathbf{u}_i - e\mathbf{u}_e$$

$$\mathbf{u}_i = \frac{\mathbf{F} \times \mathbf{B}}{qB^2} = \frac{e\mathbf{E} \times \mathbf{B}}{eB^2} = -\hat{\mathbf{x}} \frac{EB}{B^2} = -\hat{\mathbf{x}} \frac{E}{B}$$

$$\mathbf{u}_e = \frac{\mathbf{F} \times \mathbf{B}}{qB^2} = \frac{-e\mathbf{E} \times \mathbf{B}}{-eB^2} = -\hat{\mathbf{x}} \frac{EB}{B^2} = -\hat{\mathbf{x}} \frac{E}{B}$$

$$\mathbf{I} \equiv e\mathbf{u}_i - e\mathbf{u}_e = e(\mathbf{u}_i - \mathbf{u}_e) = 0$$

Blue



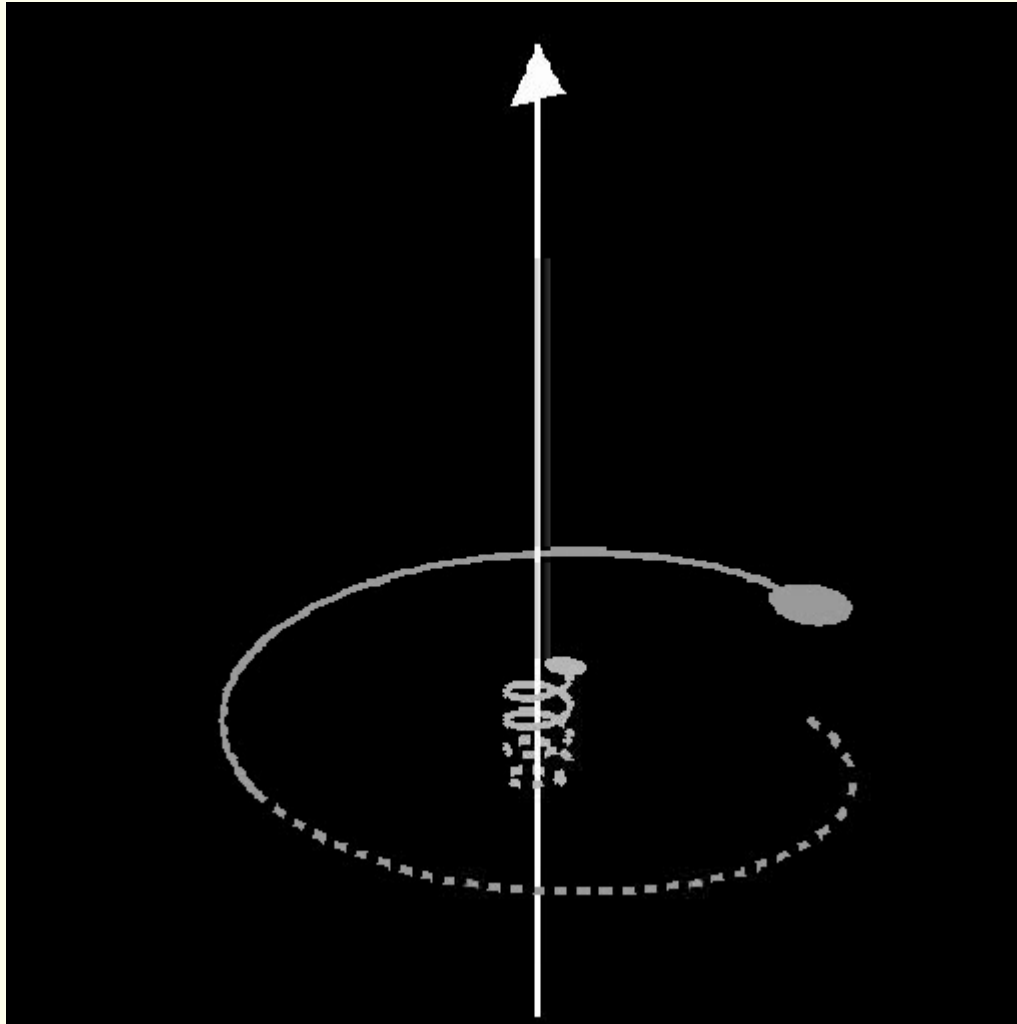
**So, if there is no current when you apply an electric field, is the conductivity of the ionospheric plasma zero ?**



What is the electron  
density at 100 km?

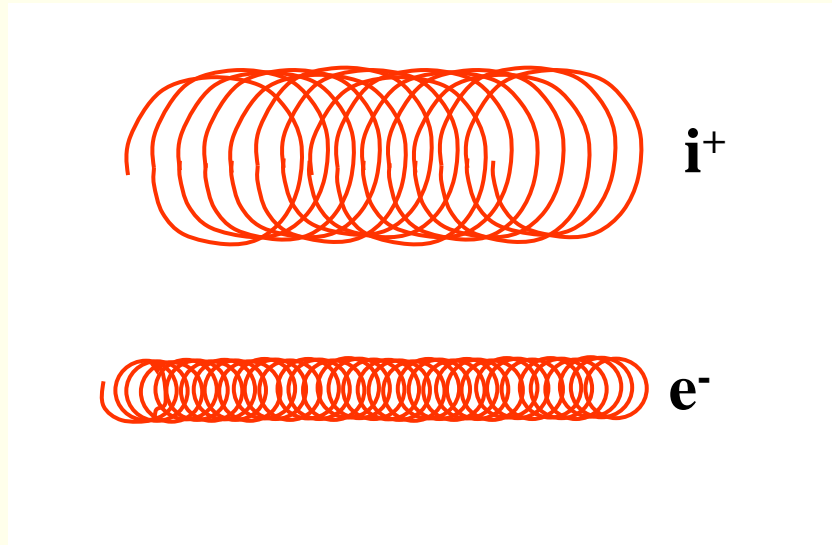
What is the neutral  
density at 100 km?

# Gyro motion

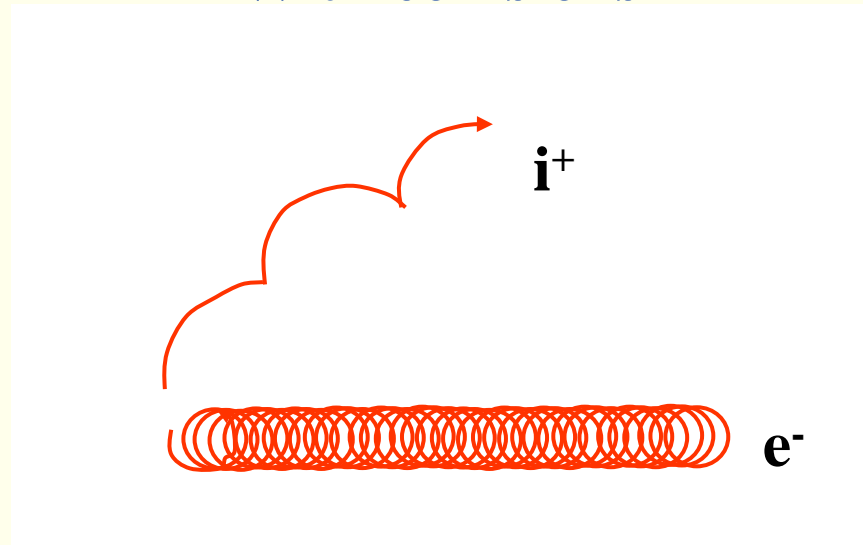


# ExB-drift

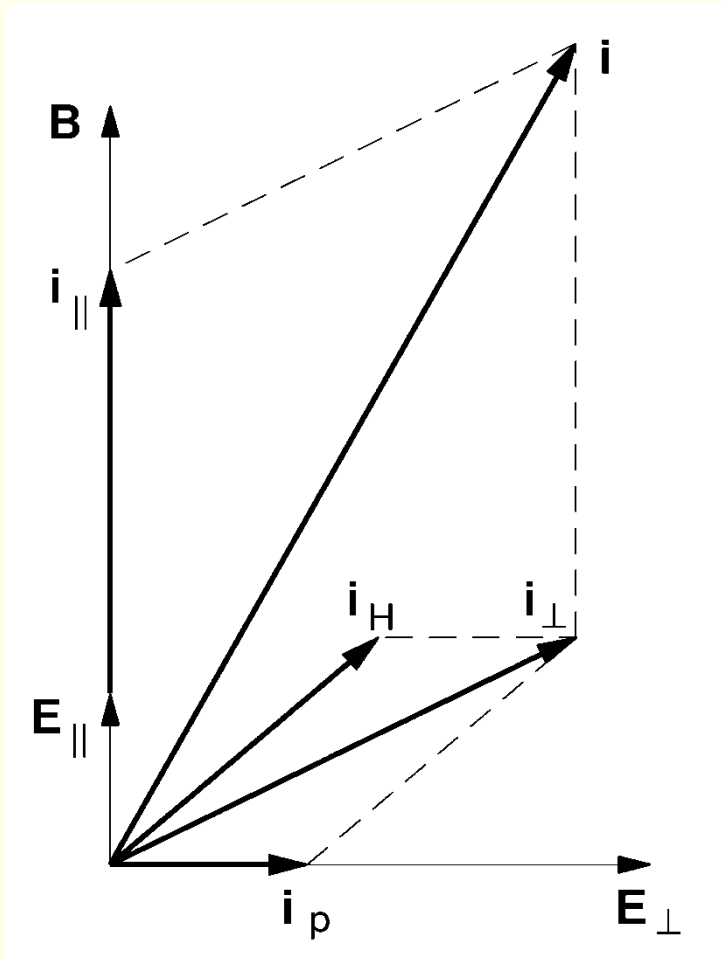
$\uparrow$   
**E**  $\odot$  **B**    **Without collisions**



**With collisions**



# Electric conductivity in a magnetized plasma



- $i_{||}$  = parallel current
- $i_p$  = Pedersen current
- $i_H$  = Hall current

# Birkeland, Hall, Pedersen



***Kristian Birkeland***

1867-1917

Norwegian  
scientist



***Edwin Hall***

1855-1938

American  
physicist



***Peder Oluf Pedersen***

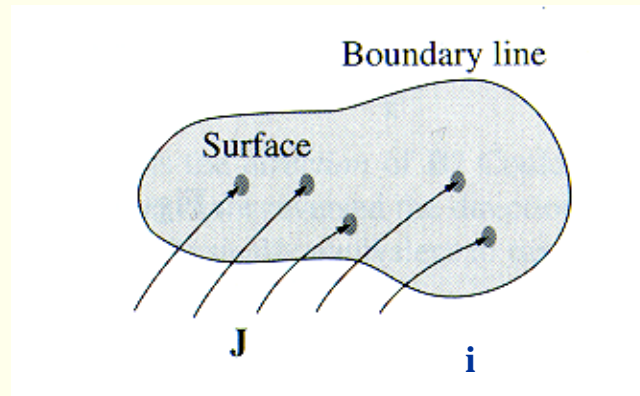
1874-1941

Danish engineer  
and physicist



# Current density

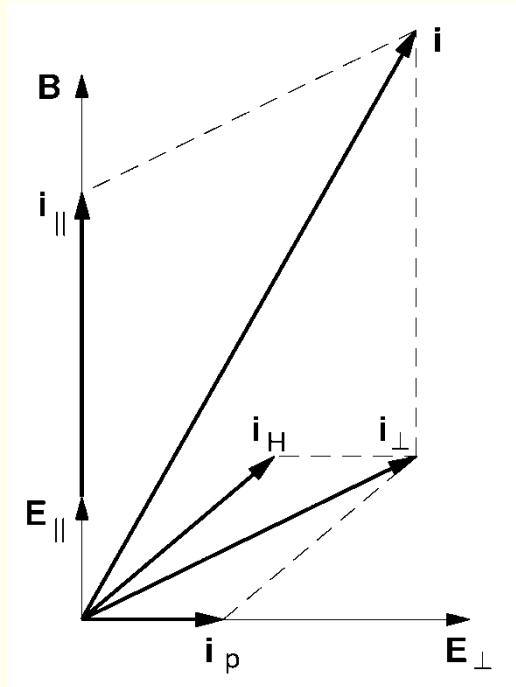
The current density  $\mathbf{j}$  is a vector field with dimension  $[\mathbf{j}] = \text{Am}^{-2}$ .



The total current  $I$  through the surface  $S$  is

$$I = \int_S \mathbf{j} \cdot d\mathbf{S}$$

# Electric conductivity in a magnetized plasma II



$$\sigma_P = \sigma_e \frac{1}{1 + \omega_{ge}^2 \tau_e^2} + \sigma_i \frac{1}{1 + \omega_{gi}^2 \tau_i^2}$$

$$\sigma_H = \sigma_e \frac{\omega_{ge} \tau_e}{1 + \omega_{ge}^2 \tau_e^2} - \sigma_i \frac{\omega_{gi} \tau_i}{1 + \omega_{gi}^2 \tau_i^2}$$

$$\sigma_{||} = \sigma_e + \sigma_i$$

$$\sigma_e = e^2 n \tau_e / m_e$$

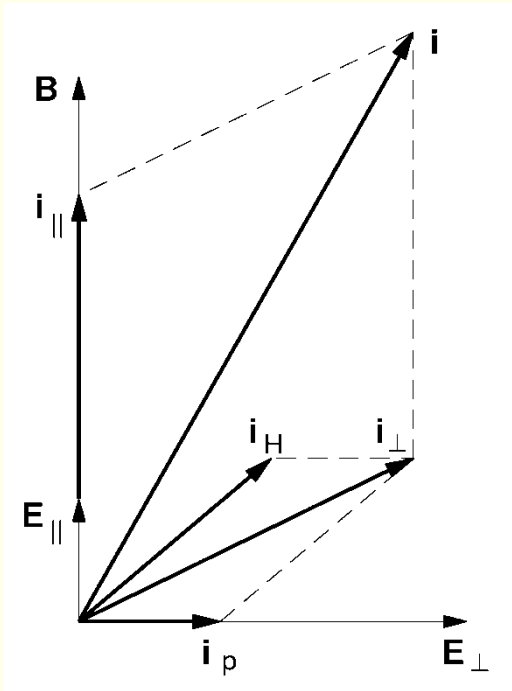
$$\sigma_i = e^2 n \tau_i / m_i$$

$$i_{||} = \sigma_{||} E_{||}$$

$$\left. \begin{aligned} i_P &= \sigma_P E_{\perp} \\ i_H &= \sigma_H E_{\perp} \end{aligned} \right\}$$

$$\text{or } \mathbf{i}_{\perp} = \sigma_P \mathbf{E}_{\perp} + \sigma_H \frac{\mathbf{B} \times \mathbf{E}_{\perp}}{B}$$

# Electric conductivity in a magnetized plasma II



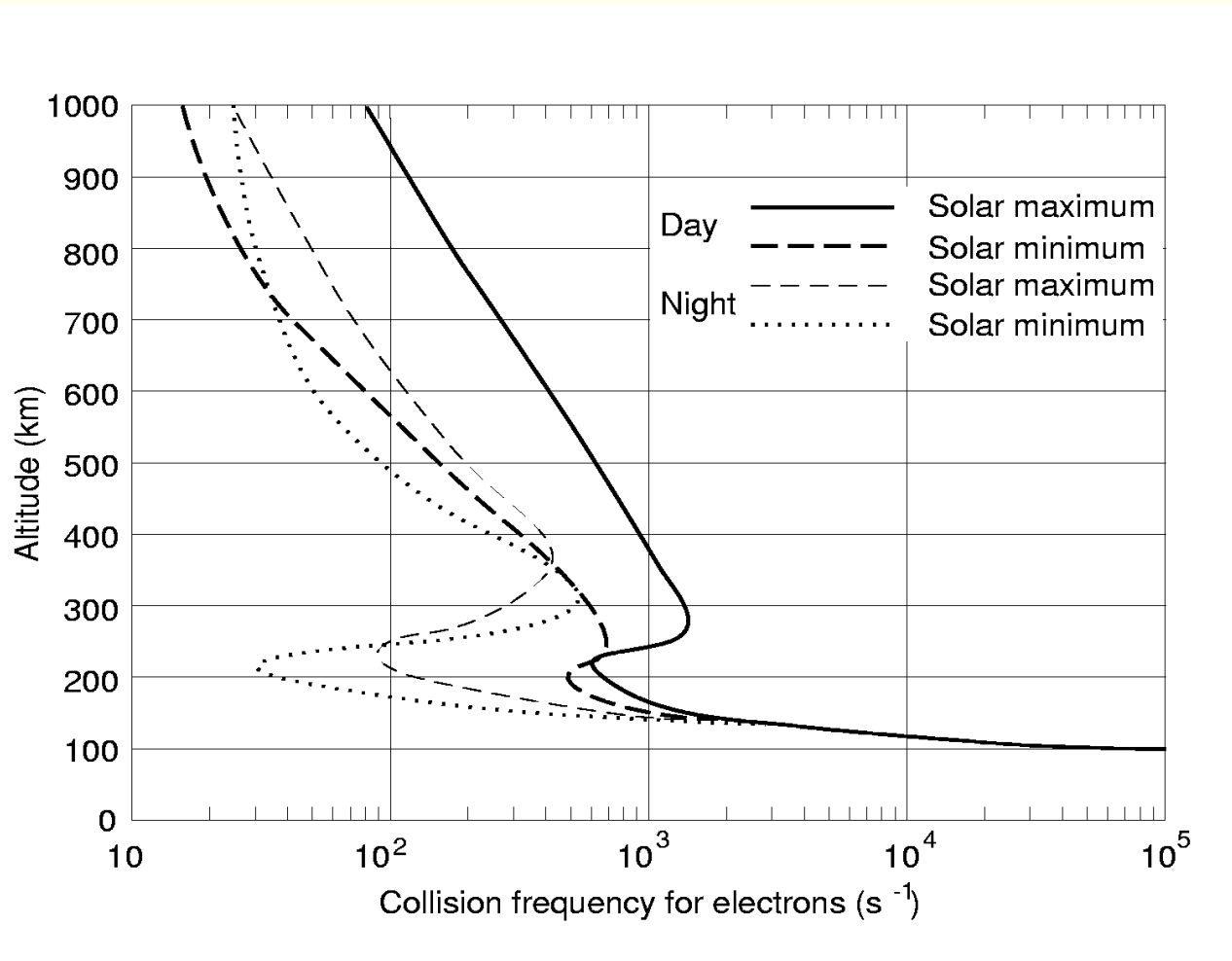
$$\mathbf{i} = \boldsymbol{\sigma} \cdot \mathbf{E}$$

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_P & \sigma_H & 0 \\ -\sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{||} \end{pmatrix}$$

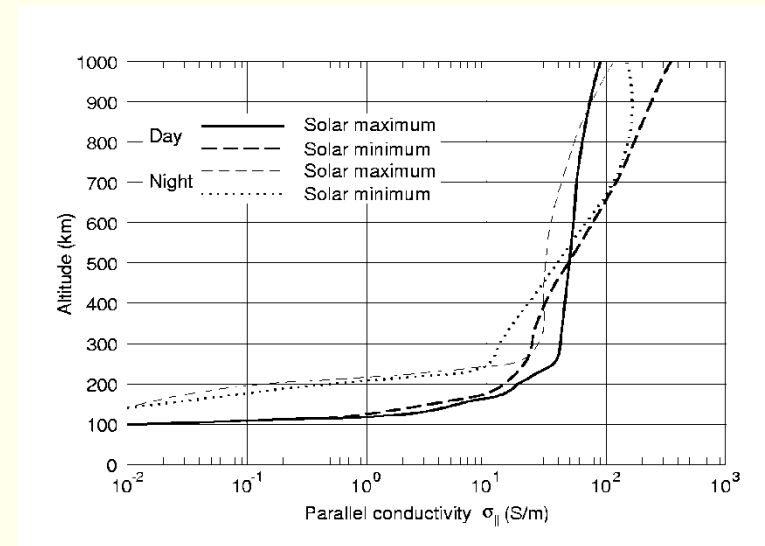
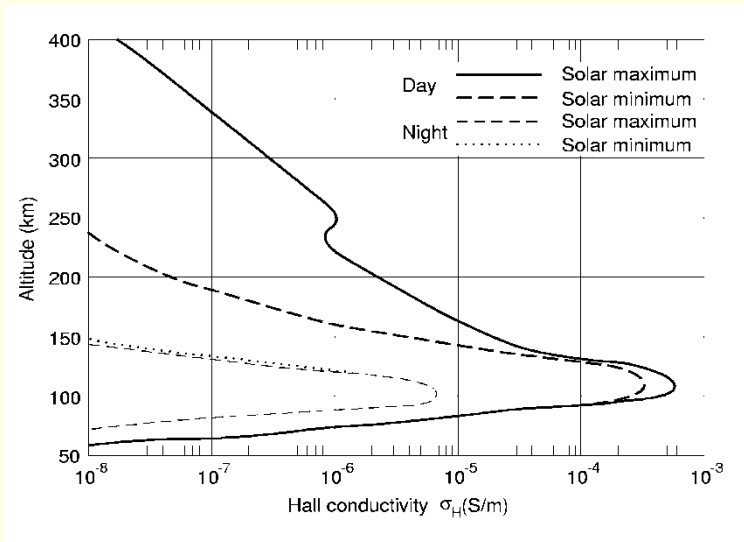
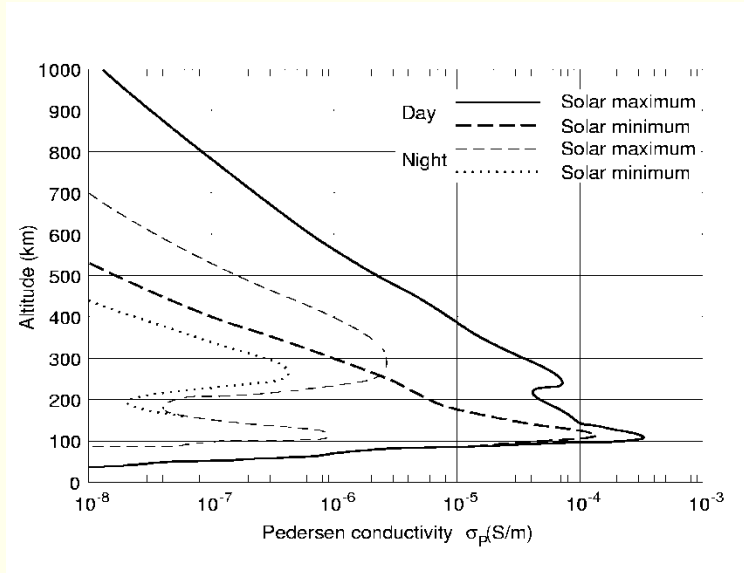
*conductivity tensor*

May be formulated as a tensor equation

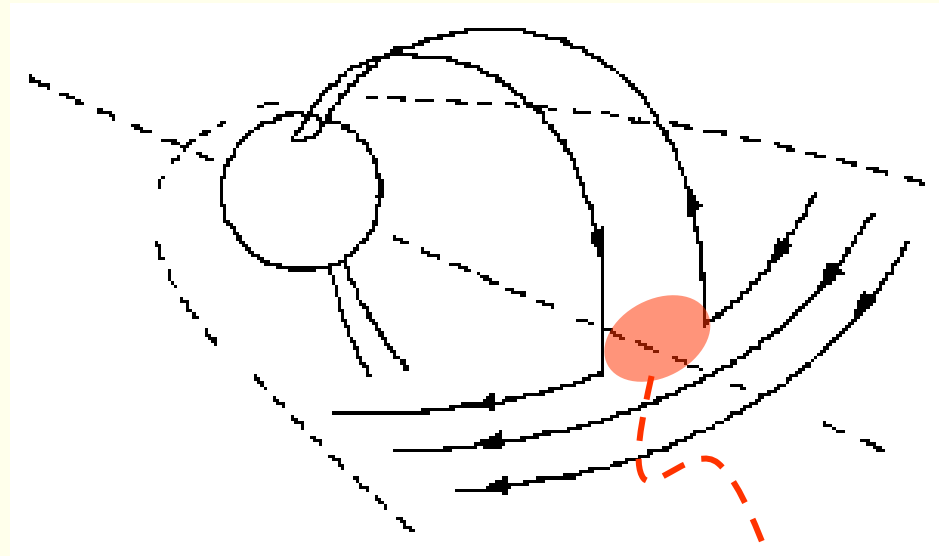
# Collisional frequency



# Ionospheric conductivities



# Consequence: Birkeland currents



Region of low conductivity

When the conductivity out in the magnetosphere is low, it is easier for the current to close through the ionosphere via currents parallel to the geomagnetic field. Such currents are called *Birkeland* currents.

# Exemple: Electric field **700 km** above the aurora.

$$\mathbf{E} = E_x \hat{\mathbf{x}} + E_y \hat{\mathbf{y}}$$

$$E_x = 1 \text{ Vm}^{-1}$$

$$E_z = 1 \text{ } \mu\text{Vm}^{-1}$$

$$\left. \begin{aligned} j_P = j_x &= 0.01 \text{ } \mu\text{Am}^{-2} \\ j_{//} = j_z &= 40 \text{ } \mu\text{Am}^{-2} \end{aligned} \right\}$$

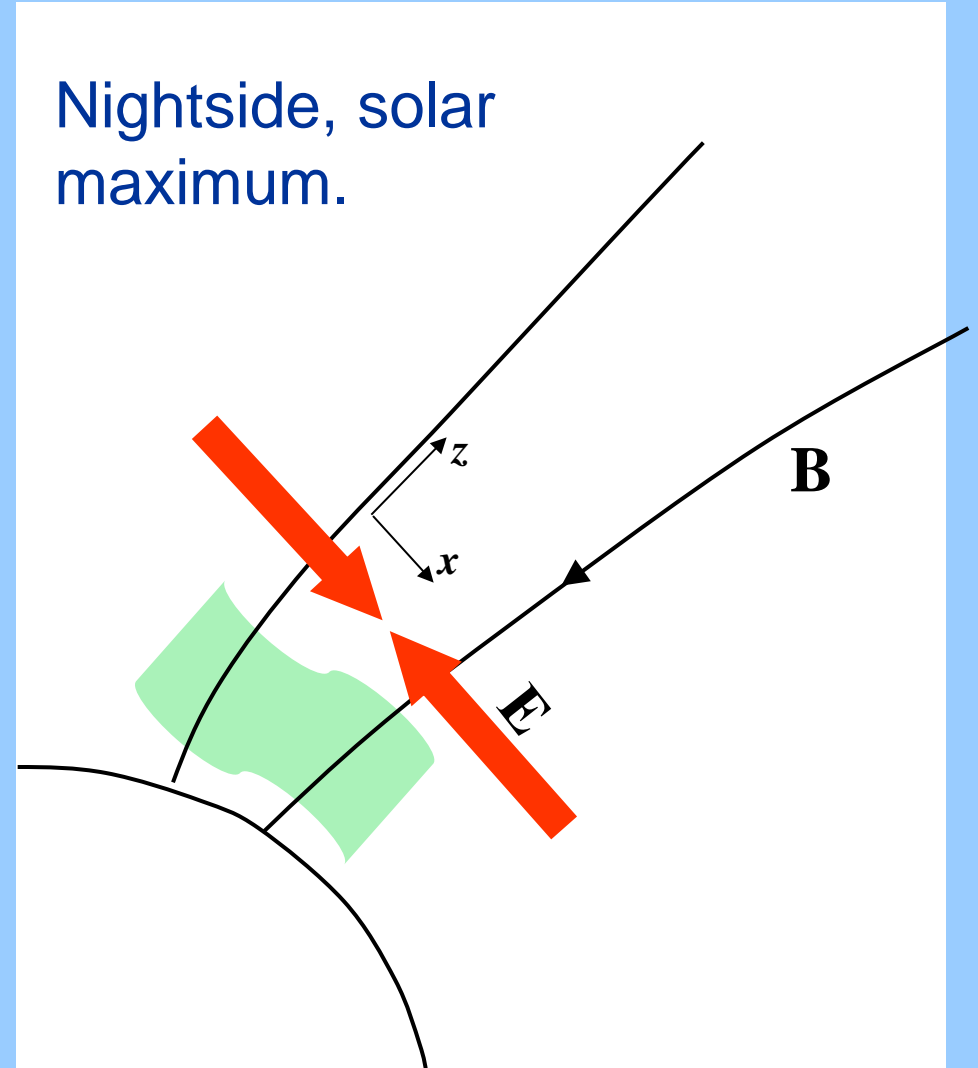
Yellow

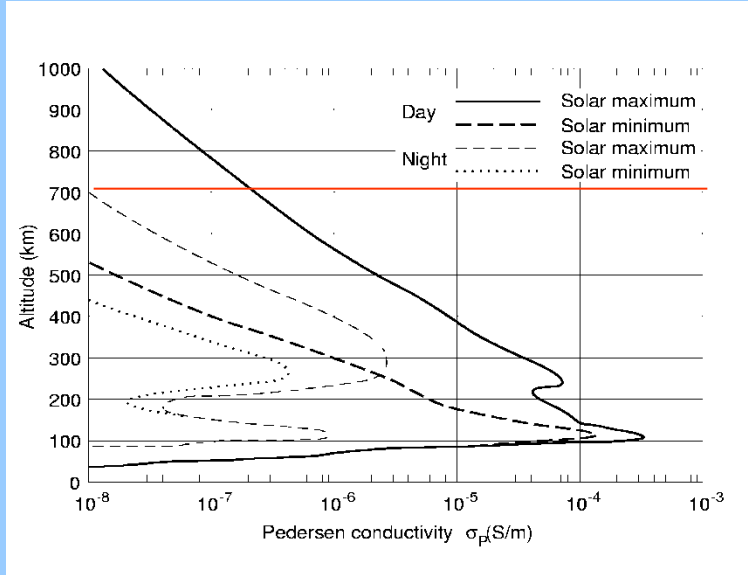
$$\left. \begin{aligned} j_P = j_x &= 10.0 \text{ } \mu\text{Am}^{-2} \\ j_{//} = j_z &= 4.0 \text{ } \mu\text{Am}^{-2} \end{aligned} \right\}$$

Red

$$\left. \begin{aligned} j_P = j_x &= 1.0 \text{ } \mu\text{Am}^{-2} \\ j_{//} = j_z &= 40 \text{ mA}\mu\text{m}^{-2} \end{aligned} \right\}$$

Blue



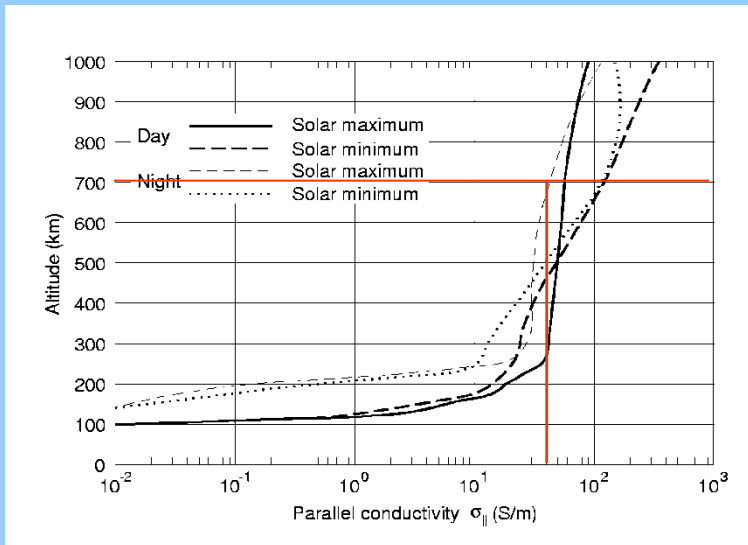


$$\sigma_P \approx 1 \cdot 10^{-8} \text{ Sm}^{-1}$$

$$\sigma_{//} \approx 40 \text{ Sm}^{-1}$$

$$j_P = j_x = \sigma_P E_x = 1 \cdot 10^{-8} \text{ Am}^{-2} = 0.01 \mu\text{Am}^{-2}$$

$$j_{//} = j_z = \sigma_{//} E_z = 40 \cdot 10^{-6} \text{ Am}^{-2} = 40 \mu\text{Am}^{-2}$$



Yellow

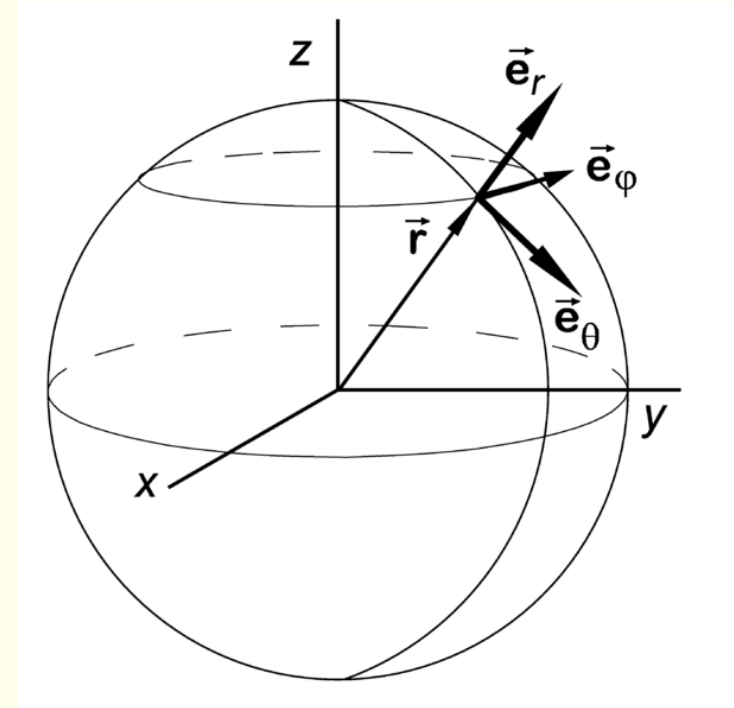
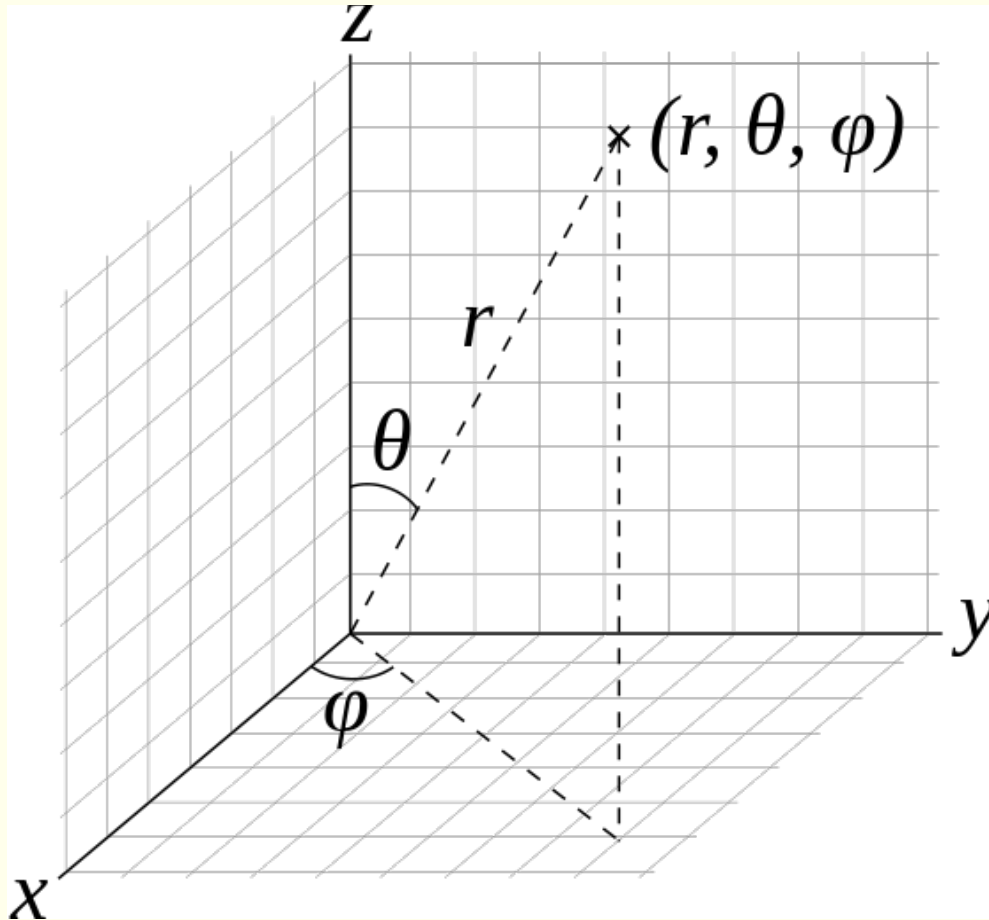




# How do we define "the magnetosphere"?

The region in space where the magnetic field is dominated by the geomagnetic field.

# Polar (spherical) coordinates



$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \arccos\left(\frac{z}{r}\right)$$

$$\varphi = \arctan\left(\frac{y}{x}\right)$$

$$x = r \sin \theta \cos \varphi$$

$$y = r \sin \theta \sin \varphi$$

$$z = r \cos \theta$$

# Geomagnetic field

Approximated by a dipole close to Earth.

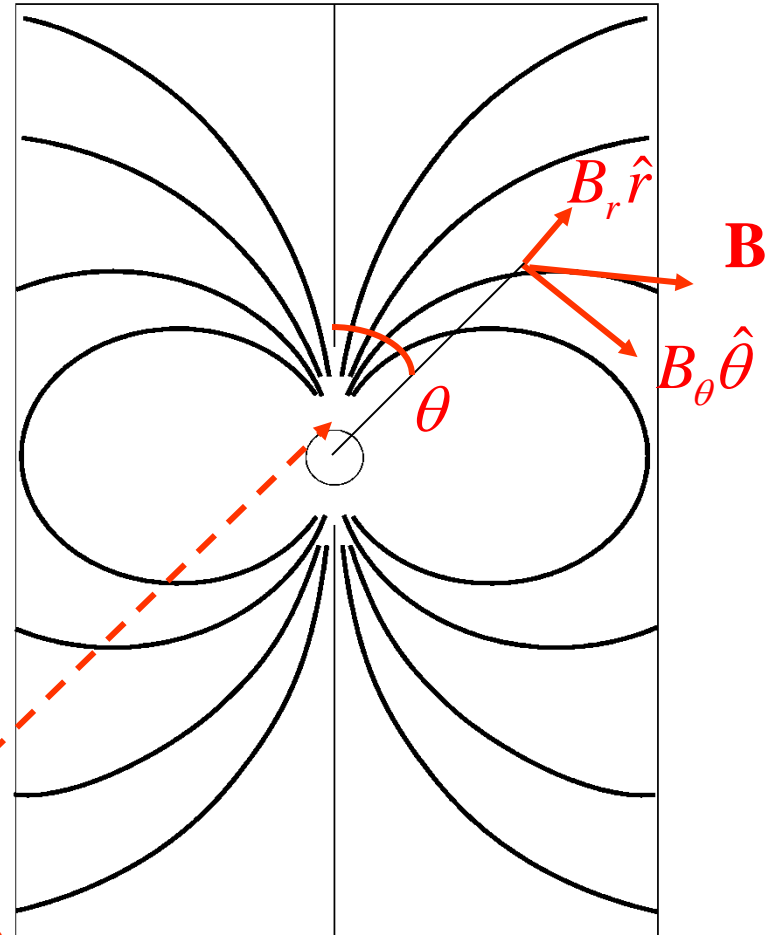
$$B_r = B_p \left(\frac{R_E}{r}\right)^3 \cos \theta$$

$$B_\theta = \frac{B_p}{2} \left(\frac{R_E}{r}\right)^3 \sin \theta$$

$$a = \frac{2\pi R_E^3 B_p}{\mu_0}$$

magnetic dipole moment

Magnetic field at the "north pole"



# Geomagnetic field

Alternative formulation of dipole field

$$B_r = B_p \left(\frac{R_E}{r}\right)^3 \cos \theta$$

$$B_\theta = \frac{B_p}{2} \left(\frac{R_E}{r}\right)^3 \sin \theta$$

$$B_r = \frac{\mu_0 a}{2\pi} \frac{1}{r^3} \cos \theta$$

$$B_\theta = \frac{\mu_0 a}{2\pi} \cdot \frac{1}{2} \cdot \frac{1}{r^3} \sin \theta$$

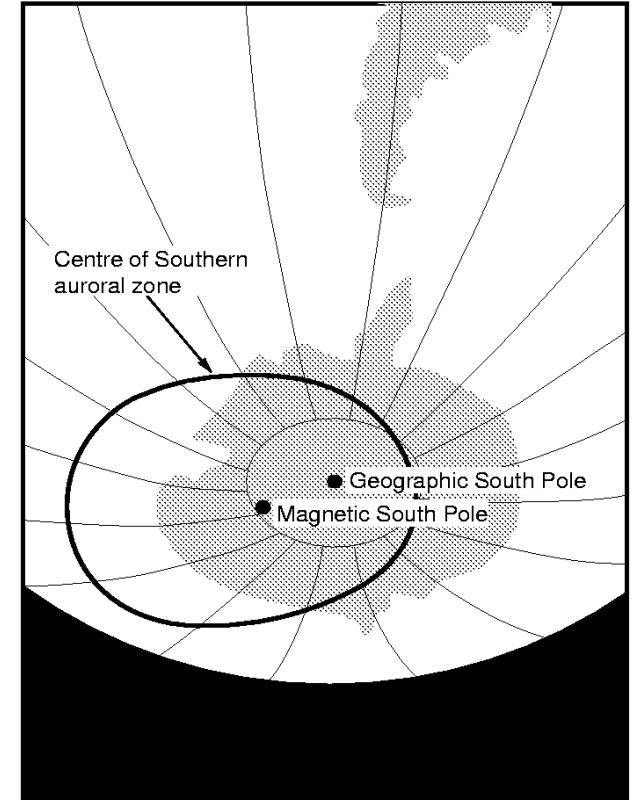
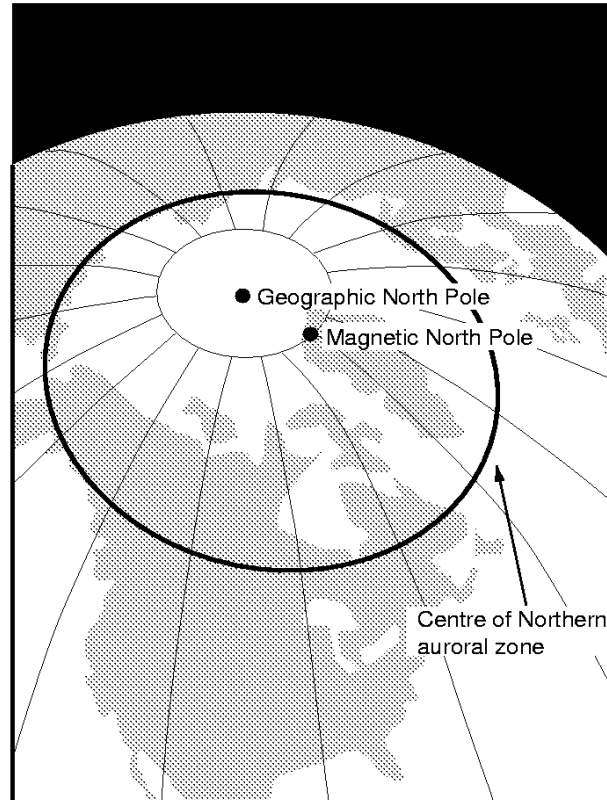
$$a = \frac{2\pi R_E^3 B_p}{\mu_0}$$

 magnetic dipole moment

# Geomagnetic field

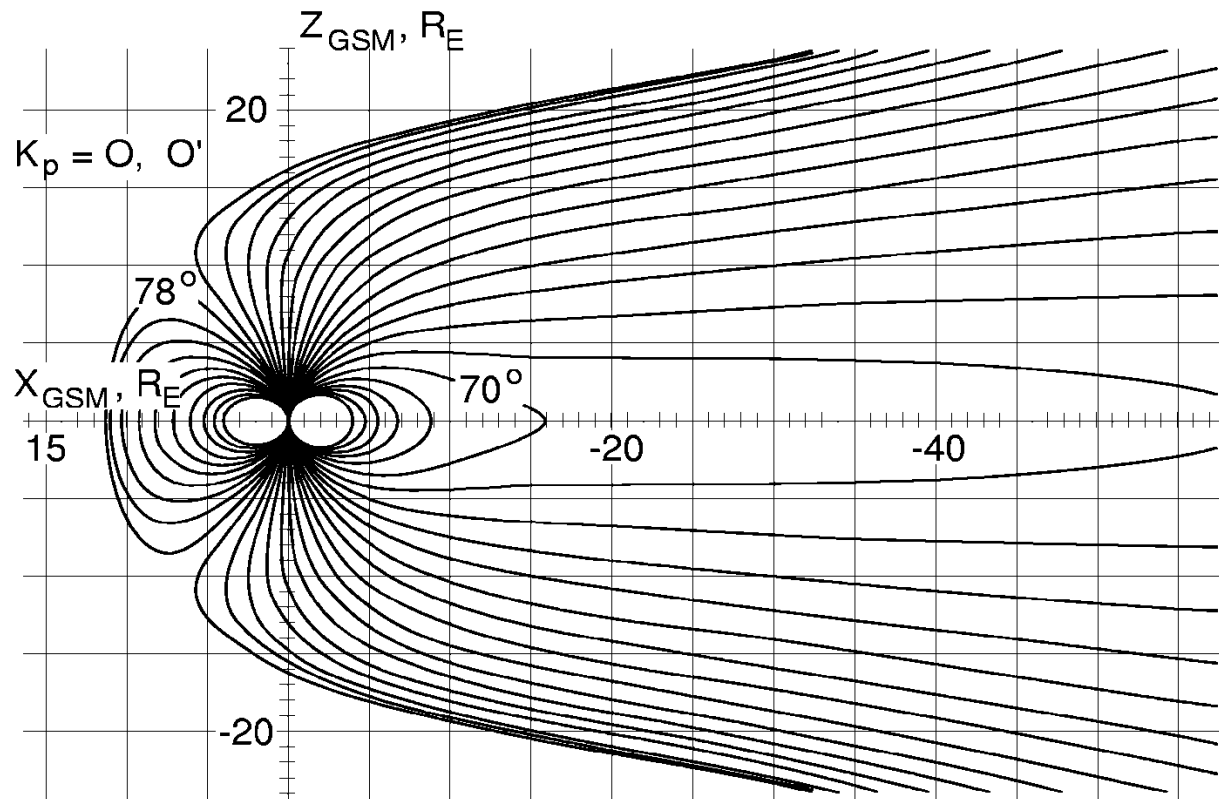
- Angle between dipole axis and spin axis:  $\approx 11^\circ$
- The geographic north pole is a magnetic south pole, and vice versa.
- $B_{equator} = 31 \mu\text{T}$ ,

$$B_{pole} = 62 \mu\text{T}$$

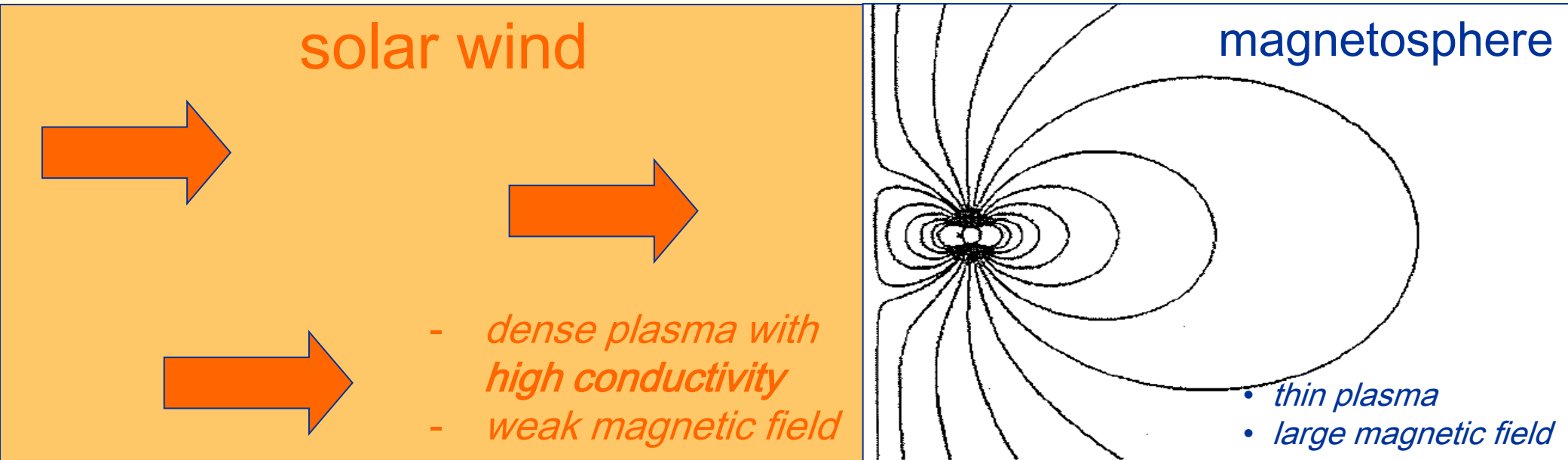


# Geomagnetic field

Modified by solar wind into tail-like configuration



# Stand-off distance from pressure balance



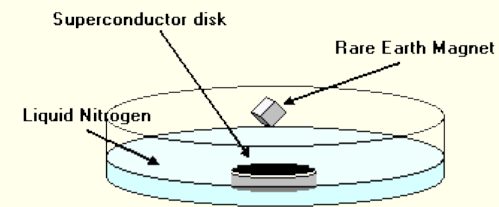
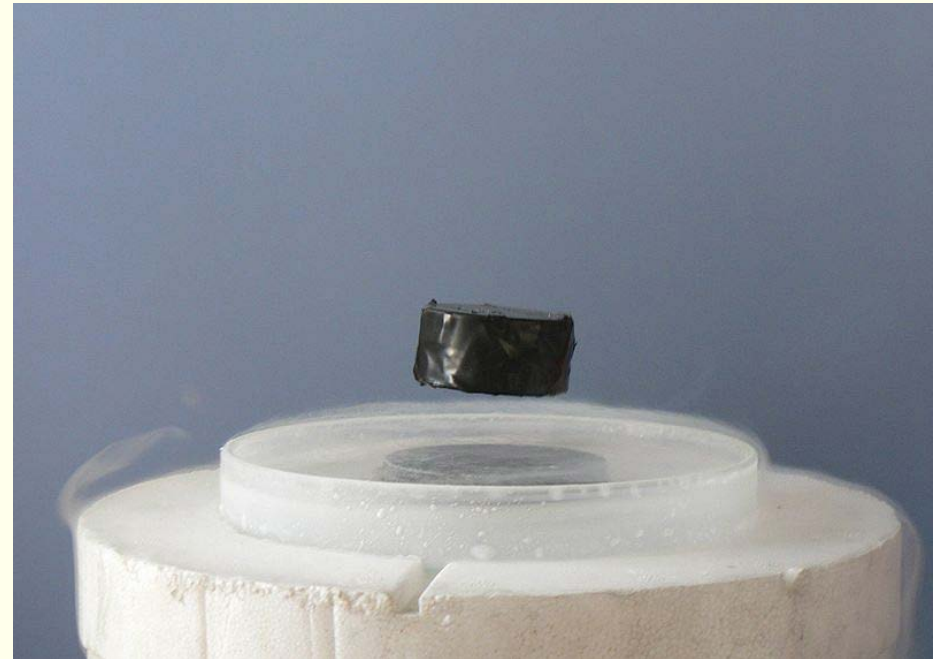
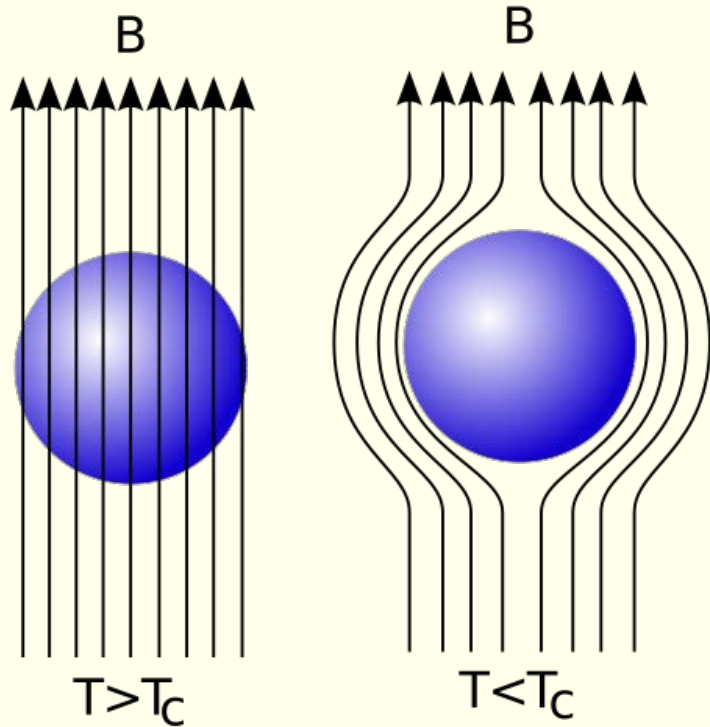
Dynamic pressure:

$$p_d = \rho_{SW} v_{SW}^2$$

Magnetic pressure:

$$p_B = \frac{B^2}{2\mu_0}$$

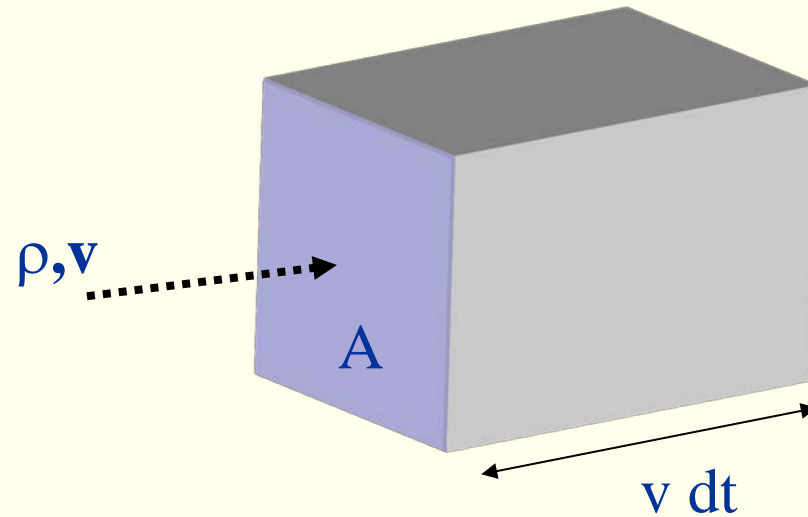
# Meissner effect in super-conductors



The Meissner Effect

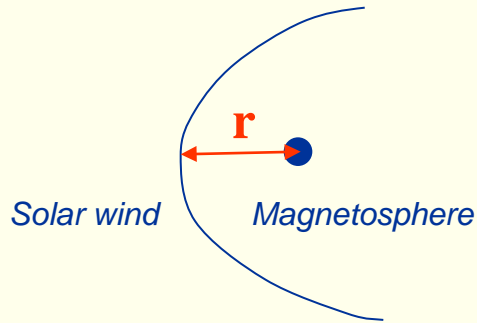


# Dynamic (kinetic) pressure



$$p_d = \frac{F}{A} = \frac{d(mv)}{dt} \frac{1}{A} \approx \frac{\Delta(mv)}{\Delta t} \frac{1}{A} = \frac{\rho \cdot Av \Delta t \cdot v}{\Delta t A} = \rho v^2$$

# Magnetopause “stand-off distance”



Dynamic pressure:  $p_d = \rho_{SW} v_{SW}^2$

Magnetic pressure:  $p_B = \frac{1}{2\mu_0} B^2$

Dipole field strength  
(in equatorial plane):  $B = \frac{\mu_0 a}{4\pi} \frac{1}{r^3}$

$$p_d = p_B \Rightarrow \rho_{SW} v_{SW}^2 = \left[ \frac{\mu_0 a}{4\pi} \frac{1}{r^3} \right]^2 / 2\mu_0 \Rightarrow$$

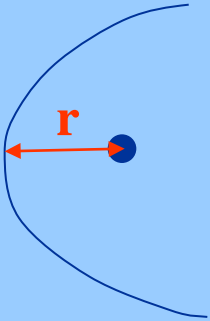
$$r = \left( \frac{\mu_0 a}{4\pi} \right)^{1/3} \left( 2\mu_0 \rho_{SW} v_{SW}^2 \right)^{-1/6}$$

$a = 8 \times 10^{22} \text{ Am}^2$ ,  $v = 500 \text{ km/s}$ ,  $\rho_{SW} = 10^7 \times 1.7 \times 10^{-27} \text{ kg/m}^3$ :

$r = 7 R_e$  (1  $R_e = 6378 \text{ km}$ )

# Standoff distance

$$v=500 \text{ km/s}, \quad \rho_{SW}=10^7 \times 1.7 \times 10^{-27} \text{ kg/m}^3: \quad \mathbf{r = 7 R_e}$$



$$r = \left( \frac{\mu_0 a}{4\pi} \right)^{1/3} \left( 2\mu_0 \rho_{SW} v_{SW}^2 \right)^{-1/6}$$

How will the standoff distance change if the magnetosphere is hit by a coronal mass ejection (CME)? ( $\rho = 10\rho_{SW}$ ,  $v = 1000 \text{ km/s}$ )

Blue

$$r = 1.8 R_e$$

Yellow

$$r = 5.8 R_e$$

Green

$$r = 3.8 R_e$$

Red

$$r = 9.8 R_e$$

# Standoff distance

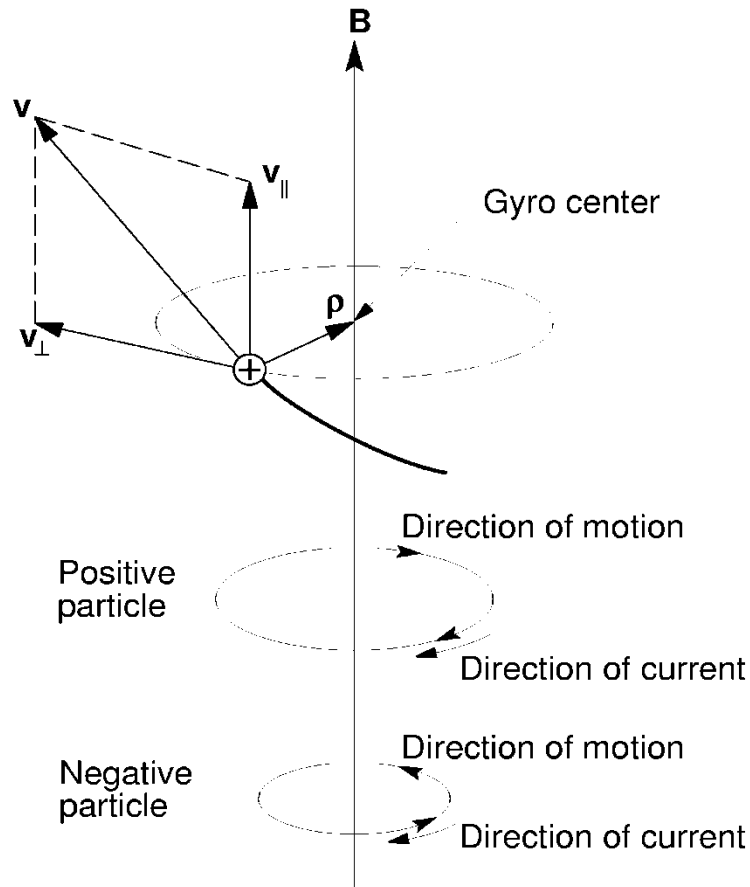
$$r = \left( \frac{\mu_0 a}{4\pi} \right)^{1/3} \left( 2\mu_0 \mathbf{10} \rho_{SW} (\mathbf{2}v)_{SW}^2 \right)^{-1/6} = \left( \frac{\mu_0 a}{4\pi} \right)^{1/3} \left( 2\mu_0 \rho_{SW} v_{SW}^2 \right)^{-1/6} \mathbf{40}^{-1/6}$$

$$40^{-1/6} \cdot 7 = 0.54 \cdot 7 = 3.8$$

Green

$$r = 3.8 R_e$$

# Particle motion in magnetic field



## gyro radius

$$\rho = \frac{mv_{\perp}}{qB}$$

## gyro frequency

$$\omega_g = \frac{qB}{m}$$

## magnetic moment

$$\mu = IA = q f_g \pi \rho^2 = mv_{\perp}^2 / 2B$$



# Adiabatic invariant

## DEFINITION:

An **adiabatic invariant** is a property of a physical system which stays constant when changes are made slowly.

*By 'slowly' in the context of charged particle motion in magnetic fields, we mean much slower than the gyroperiod.*

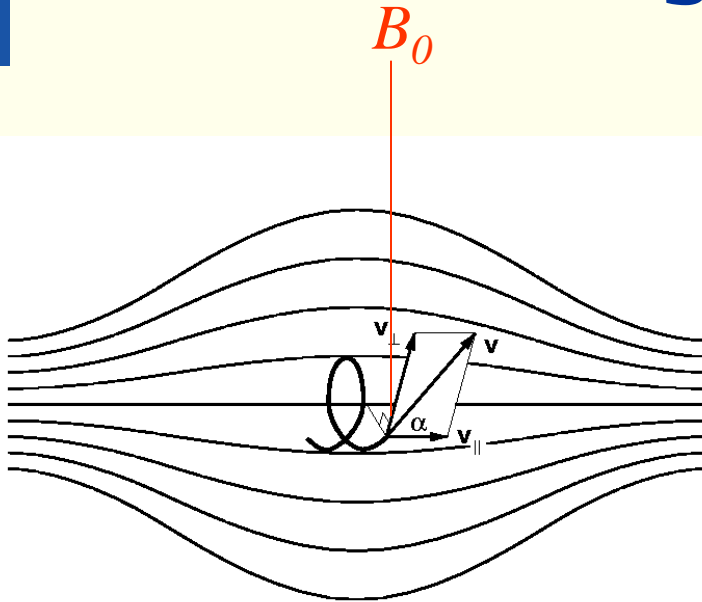
'First adiabatic invariant' of particle drift:

$$\mu = \frac{mv_{\perp}^2}{2B}$$

# Magnetic mirror

$mv^2/2$  constant (energy conservation) 

$$\frac{\sin^2 \alpha}{B} = \textit{konst}$$



The magnetic moment  $\mu$  is an *adiabatic invariant*.

$$\mu = \frac{mv_{\perp}^2}{2B} = \frac{mv^2 \sin^2 \alpha}{2B}$$

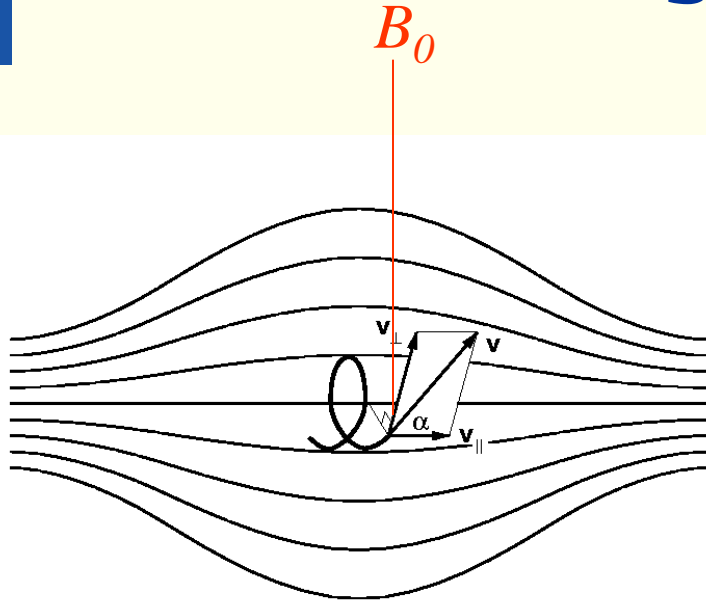
Red

$\alpha$  increases

Yellow

$\alpha$  decreases

# Magnetic mirror



$mv^2/2$  constant (energy conservation)  $\rightarrow$

$$\frac{\sin^2 \alpha}{B} = \textit{konst}$$

What happens with  $\alpha$  as the particle moves into the stronger magnetic field?

$$\sin \alpha = \sqrt{B \cdot \textit{konst}}$$

Red

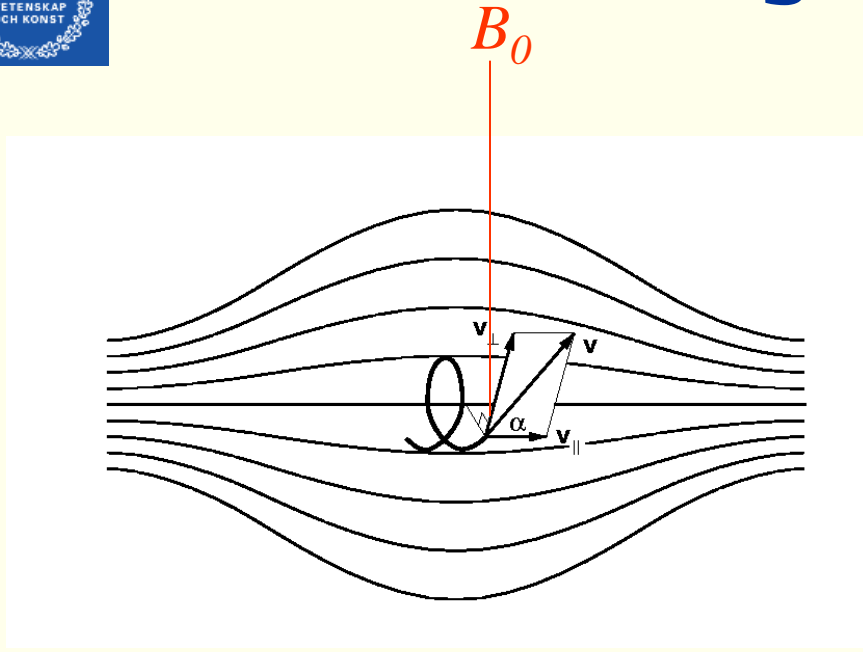
$\alpha$  increases

The magnetic moment  $\mu$  is an *adiabatic invariant*.

$$\mu = \frac{mv_{\perp}^2}{2B} = \frac{mv^2 \sin^2 \alpha}{2B}$$



# Magnetic mirror



The magnetic moment  $\mu$  is an *adiabatic invariant*.

$$\mu = \frac{mv_{\perp}^2}{2B} = \frac{mv^2 \sin^2 \alpha}{2B}$$

$mv^2/2$  constant (energy conservation)  $\rightarrow$

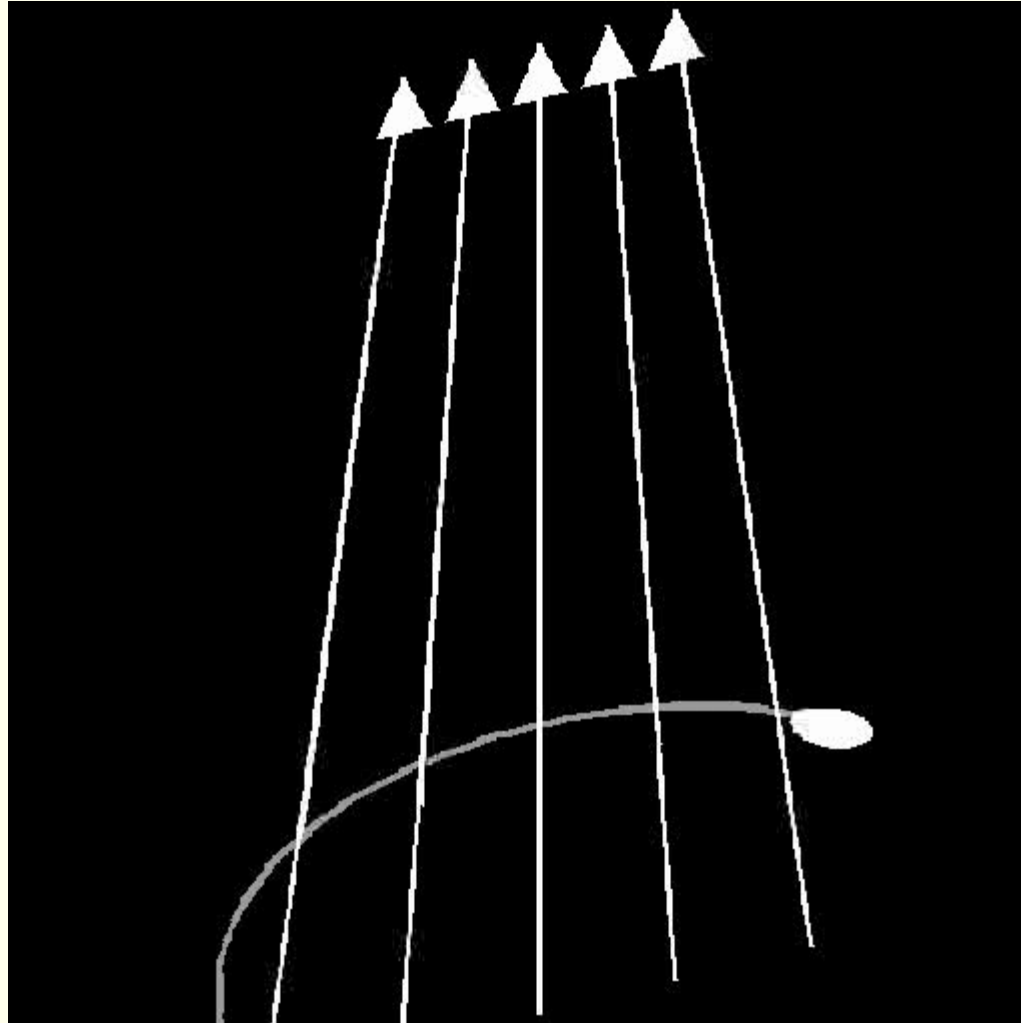
$$\frac{\sin^2 \alpha}{B} = \text{konst}$$

particle turns when  $\alpha = 90^\circ$   $\rightarrow$

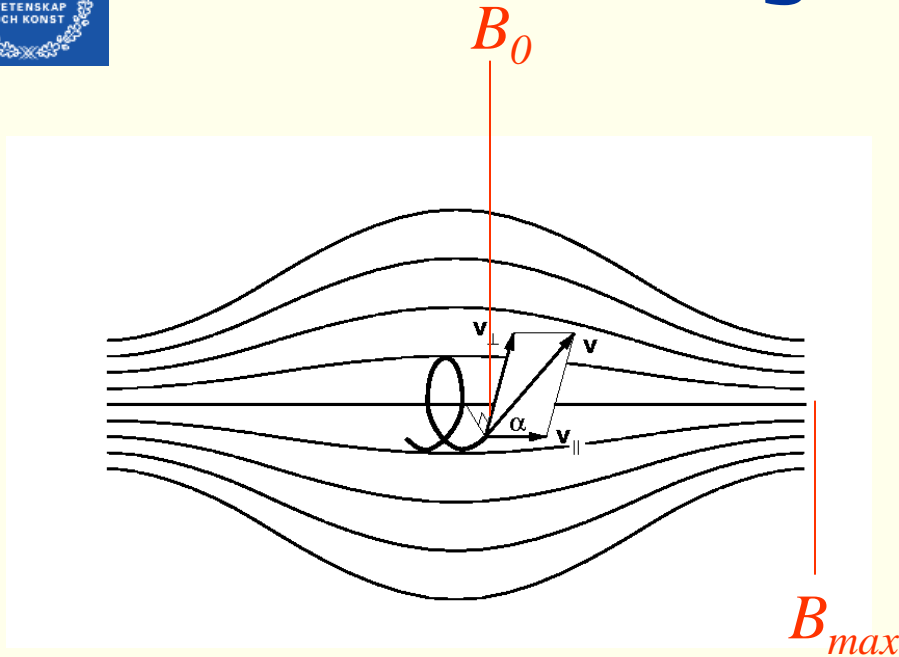
$$\frac{B_{\text{turn}}}{\sin^2 90^\circ} = \frac{B_0}{\sin^2 \alpha} \rightarrow$$

$$B_{\text{turn}} = \frac{B_0}{\sin^2 \alpha}$$

# Magnetic mirror



# Magnetic mirror



$mv^2/2$  constant (energy conservation) →

$$\frac{\sin^2 \alpha}{B} = \text{konst}$$

particle turns when  $\alpha = 90^\circ$  →

$$B_{\text{turn}} = B_0 / \sin^2 \alpha$$

If maximal  $B$ -field is  $B_{\text{max}}$  a particle with pitch angle  $\alpha$  can only be turned around if

$$B_{\text{turn}} = B_0 / \sin^2 \alpha \leq B_{\text{max}} \rightarrow$$

$$\alpha > \alpha_{lc} = \arcsin \sqrt{B_0 / B_{\text{max}}}$$

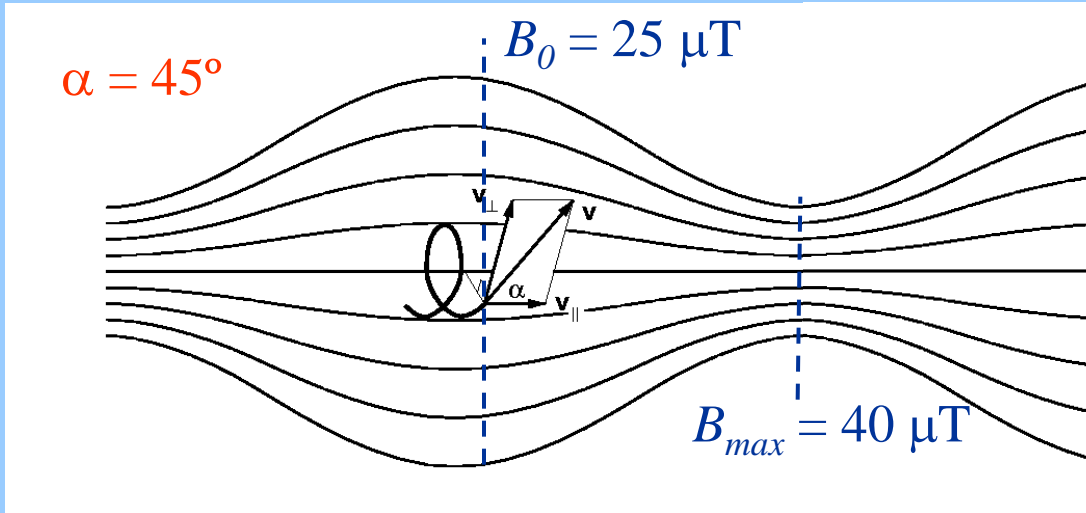
Particles in  
loss cone :

$$\alpha < \alpha_{lc}$$

The magnetic moment  $\mu$  is an  
*adiabatic invariant*.

$$\mu = \frac{mv_{\perp}^2}{2B} = \frac{mv^2 \sin^2 \alpha}{2B}$$

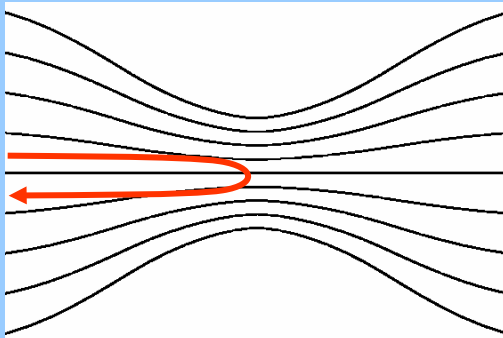
# What will happen to the particle?



$$\alpha_{lc} = \arcsin \sqrt{B_0 / B_{max}}$$

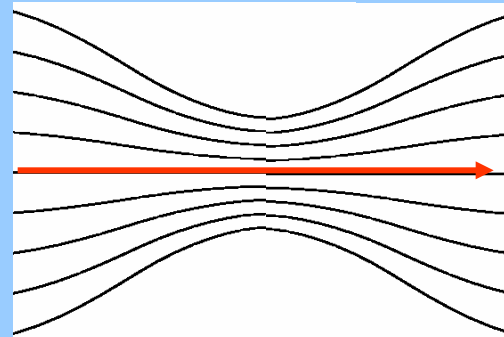
Blue

It will mirror

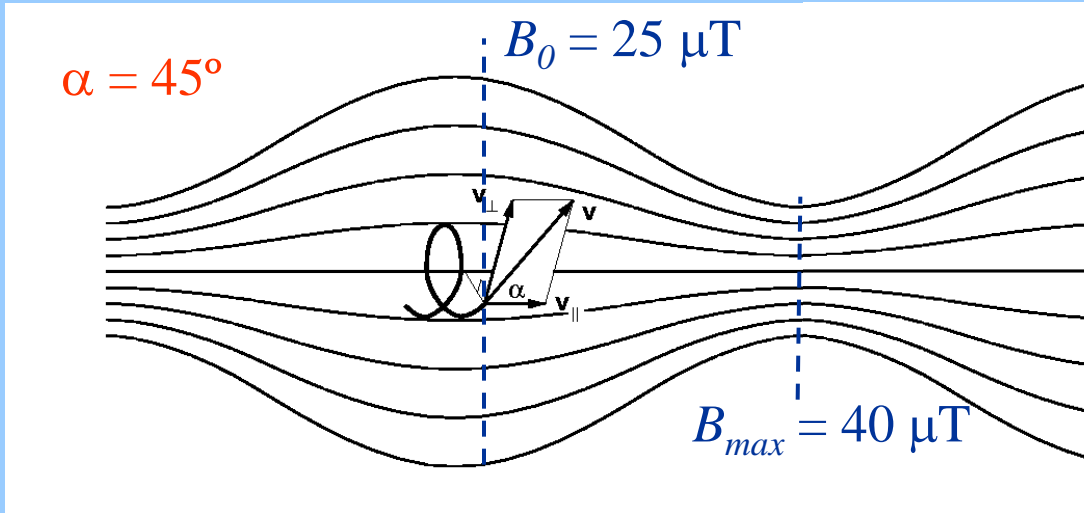


Yellow

It will escape



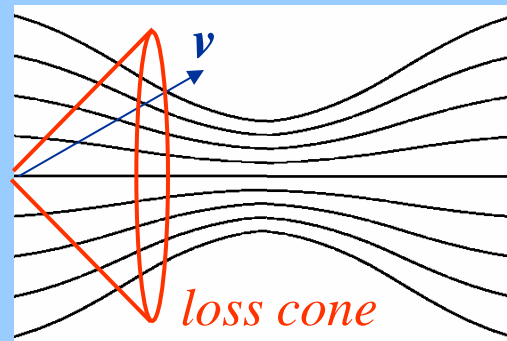
# What will happen to the particle?



$$\alpha_{lc} = \arcsin \sqrt{B_0 / B_{max}} =$$

$$\arcsin \sqrt{25 / 40} = 52^\circ$$

**Yellow** It will escape





# Last Minute!



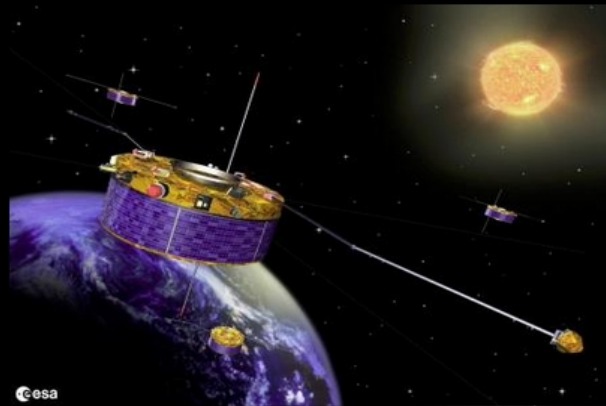
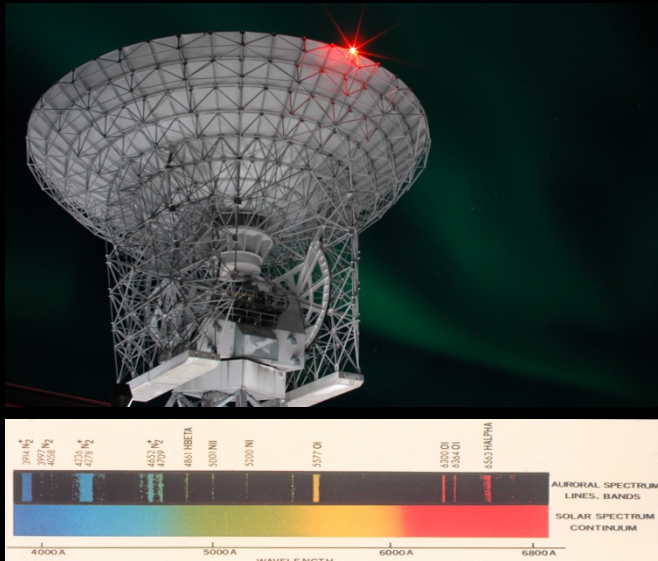
# Last Minute!

- What was the most important thing of today's lecture? Why?
- What was the most unclear or difficult thing of today's lecture, and why?
- Other comments

# Courses at the Alfvén Laboratory

## EF2230 EXPERIMENTAL METHODS IN SPACE PLASMA PHYSICS , 6 ECTS credits, period 2

- operation principles of experimental techniques in space plasma physics
- interpretation of measurements
- technical implementations of various measurements techniques
- identify major limitations, and order of magnitude estimate of performance



### Hands-on:

- Critical analysis and oral presentation
- Data acquisition or analysis using commercial software